**VOLK Geometry**

**Unit 11: SOL Geometry Crash Course**

Mr. Andrew Volk

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**Student Name:**_____

**Block:**_____

2

## SOL G.1

The student will construct and judge the validity of a logical argument consisting of a set of premises and a
conclusion. This will include
a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form;
c) using Venn diagrams to represent set relationships; and
d) using deductive reasoning.

- Identify the converse, inverse, and contrapositive of a conditional statement.
- Translate verbal arguments into symbolic form, such as (p → q) and (~p → ~q).
- Determine the validity of a logical argument.
- Use valid forms of deductive reasoning, including the law of syllogism, the law of the contrapositive, the law of detachment, and counterexamples.
- Select and use various types of reasoning and methods of proof, as appropriate.
- Use Venn diagrams to represent set relationships, such as intersection and union.
- Interpret Venn diagrams.
- Recognize and use the symbols of formal logic, which include →, ↔, ~, ∴, ∧, and ∨.

### WHAT I NEED TO KNOW:

#### CONDITIONAL STATEMENTS

| converse | inverse | contrapositive |
|---|---|---|
|  |  |  |

**WHICH STATEMENT IS LOGICALLY EQUIVALENT TO THE CONDITIONAL?**

#### LAWS

| 1. | 2. | 3. |
|---|---|---|
|  |  |  |

#### Symbols

| → |  |
|---|---|
| ↔ |  |
| ~ |  |
| ∴ |  |
| ∧ |  |
| ∨ |  |

#### Venn diagrams

| If p, then q. | ALL | SOMETIMES | NO | UNION OF 2 SETS | INTERSECTION OF 2 SETS |
|---|---|---|---|---|---|
|  |  |  |  |  |  |

| COUNTEREXAMPLE: |  |
|---|---|

## G.1 PROBLEMS:

Let $m$ represent:

    *Angle A is obtuse.*

Let $n$ represent:

    *Angle B is obtuse.*

| A. | B. | C. | D. |
|---|---|---|---|
| $m \to n$ | $m \to n$ | $m \leftrightarrow n$ | $m \leftrightarrow n$ |
| $m \wedge n$ | $m \vee n$ | $m \wedge n$ | $m \vee n$ |
| $\therefore m \vee n$ | $\therefore m \wedge n$ | $\therefore m \vee n$ | $\therefore m \wedge n$ |

**Which is a symbolic representation of the following argument?**

    *Angle A is obtuse if and only if Angle B is obtuse.*

    *Angle A is obtuse or Angle B is obtuse.*

    *Therefore, Angle A is obtuse and Angle B is obtuse.*

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The Venn diagram represents the set of cell phones in a store.
- Let P represent the cell phones that take pictures.
- Let I represent the cell phones that connect to the Internet.
- Let G represent the cell phones that have games.

Identify each region of the Venn diagram that represents the cell phones that only take pictures and have games.



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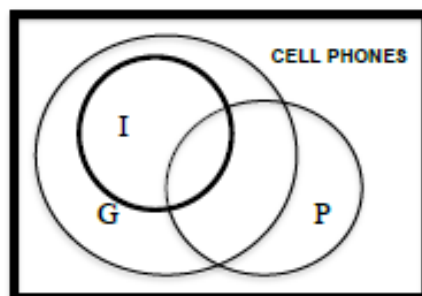
Let $p$ represent

    *$\angle A$ is acute.*

Let $q$ represent

    *$\angle B$ is acute.*

Create a symbolic representation of the following argument.

| | |
|---|---|
| $\angle A$ is acute if and only if $\angle B$ is acute. | |
| $\angle A$ is acute or $\angle B$ is acute. | |
| Therefore, $\angle A$ is acute and $\angle B$ is acute. | |

| | | | | | |
|---|---|---|---|---|---|
| $p \to q$ | $p \leftrightarrow q$ | $p \wedge q$ | $p \vee q$ | $\therefore p \wedge q$ | $\therefore p \vee q$ |

4

**(1)** Which statement is the converse of the following statement?

**If two angles are complementary, then the sum of their angle measures is 90°.**

**A** If two angles are not complementary, then the sum of their angle measures is not 90°.

**B** Two angles are complementary if and only if the sum of their angle measures is 90°.

**C** If the sum of the measures of two angles is not 90°, then the angles are not complementary.

**D** If the sum of the measures of two angles is 90°, then the angles are complementary.

**(2)** *If two triangles are both equilateral, then they are similar.*

**Which of the following best describes the *contrapositive* of the assertion above?**

**A** If two triangles are not both equilateral, then they are not similar.

**B** Two triangles are similar if and only if they are both equilateral.

**C** If two triangles are not similar, then they are not both equilateral.

**D** If two triangles are similar, then they are both equilateral.

**(3)** You are told that a conditional statement is false. Which statement is also false?

**F** inverse

**G** contrapositive

**H** converse

**J** conclusion

**(4)** "Switch the hypothesis and the conclusion." This is a procedure for constructing which of the following?

**A** The inverse

**B** The converse

**C** The contrapositive

**D** None of the above

**(5)** What is the inverse of the given statement?

**GIVEN: If you do not enter the contest, you cannot win the contest.**

**F** If you enter the contest, you can win the contest.

**G** If you cannot win the contest, do not enter the contest.

**H** If you enter the contest, you cannot win the contest.

**J** If you can win the contest, then enter the contest.

**(6)** Given that the inverse of a statement is true, what other statement is true?

**F** The original statement

**G** The hypothesis

**H** The converse

**J** The contrapositive

**(7)** If an argument containing a conditional statement among the premises is valid, then the argument remains valid when the conditional statement is replaced with which of the following?

**A** Its negation

**B** Its inverse

**C** Its converse

**D** Its contrapositive

**(8)** If a conditional statement seems difficult to prove, one can instead try to prove which equivalent statement?

**F** The negation

**G** The inverse

**H** The converse

**J** The contrapositive

**9** Let *p* be "it is raining," let *q* be "it is thundering," and let *r* be "we cannot swim." What is ~*r* → ~*q*?

A If it is thundering, then we cannot swim.

B If we can swim, then it is not thundering.

C If we cannot swim, then it is not thundering.

D If we can swim, then it is thundering.

**10** Which of the following argument forms is *invalid*?

A *q*
  *p* → *q*
  Therefore, *p*

B *p*
  *p* → *q*
  Therefore, *q*

C ~*q*
  *p* → *q*
  Therefore, ~*p*

D *p* → *q*
  *q* → *r*
  Therefore, *p* → *r*

**11** Which of the following argument forms is *valid*?

F *p* → *q*
  ~*p*
  Therefore, ~*q*

G *p* → *q*
  ~*q*
  Therefore, ~*p*

H *p* → *q*
  *q*
  Therefore, *p*

J *p* → *q*
  *p* → *r*
  Therefore, *q* → *r*

**12** Let *p* be "it is raining," let *q* be "it is thundering," and let *r* be "we cannot swim." The statement ~*p* → ~*r* could be

F The inverse of *r* → *p*

G The inverse of ~*r* → ~*p*

H The contrapositive of *r* → *p*

J The converse of *r* → *p*

**13** Which of the following is guaranteed to make a conditional statement true?

A A true hypothesis

B A false hypothesis

C A false conclusion

D None of the above

**14** Let *p* be "the election was stolen," let *q* be "the ballots were tampered with," and let *r* be "officials lost control of the ballots." Translate the following argument into symbolic form.

*If officials did not lose control of the ballots, then ballots were not tampered with.*

*And if the ballots were not tampered with, then the election was not stolen.*

*Therefore, if the election was stolen, then officials lost control of the ballots.*

A ~*r* → ~*q*
  ~*q* → ~*p*
  Therefore, *p* → *r*

B ~*p* → ~*q*
  ~*q* → ~*r*
  Therefore, *r* → *p*

C ~*q* → *r*
  *r* → ~*p*
  Therefore, *p* → *q*

D *r* → ~*p*
  *p* → ~*q*
  Therefore, *q* → ~*r*

**15** Which of the following is guaranteed to make a conditional statement false?

F A true hypothesis

G A false hypothesis

H A false conclusion

J None of the above

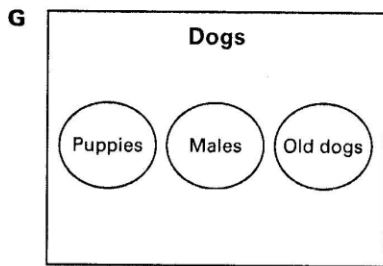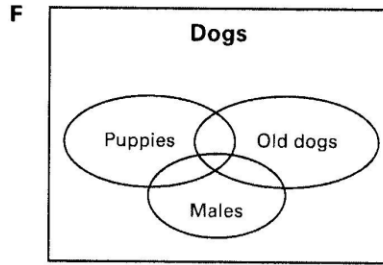**16** Which of the following statements is represented by the Venn diagram?

**Animals**
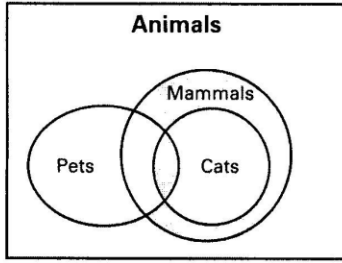


**A** No pets are mammals.

**B** All cats are mammals.

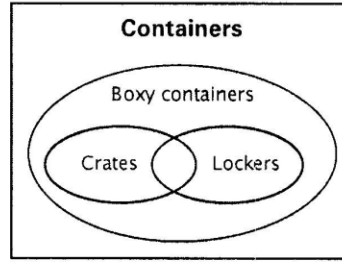**C** All cats are pets.

**D** All mammals are cats.

**17** Some puppies are males.
Some males are old dogs.

Therefore, some puppies are old dogs.

Which of the following Venn diagrams shows the above argument to be invalid?

**F**

**Dogs**



**G**

**Dogs**



**H**

**Dogs**



**J**

**Dogs**
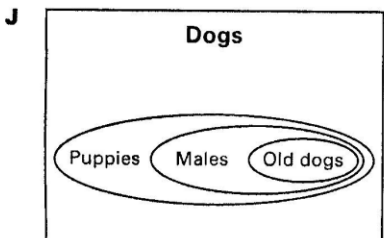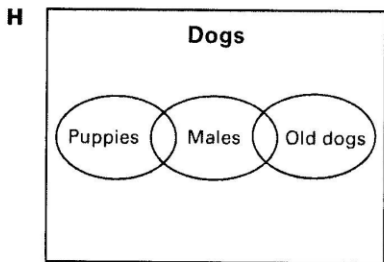


**18** Which of the following statements is *not* represented by the Venn diagram?

**Containers**



**F** No boxy containers are crates.

**G** All lockers are boxy containers.

**H** Some crates are lockers.

**J** Some boxy containers are lockers.

**19** Which of the following Venn diagrams represents the statement "All jackets are outerwear"?

**A**

**Clothing**



**B**

**Clothing**



**C**

**Clothing**



**D**

**Clothing**
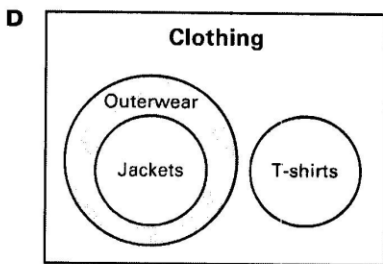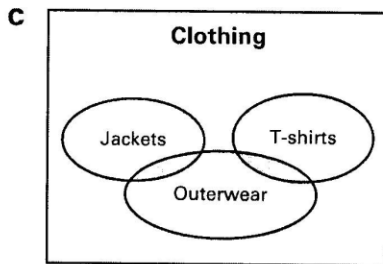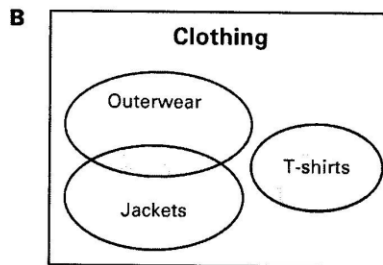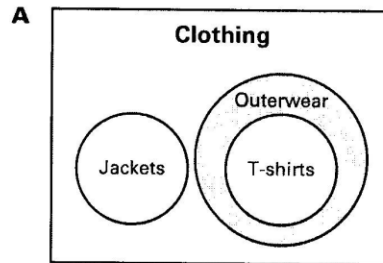
| SOL G.2 |
|---|

The student will use the relationships between angles formed by two lines cut by a transversal to
  a) determine whether two lines are parallel;
  b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs;
  c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

• Use algebraic and coordinate methods as well as deductive proofs to verify whether two lines are parallel.
• Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.
• Solve real-world problems involving intersecting and parallel lines in a plane.

## WHAT I NEED TO KNOW:

| Special ANGLE PAIR NAMES | WHERE THEY ARE LOCATED on parallel lines cut by a transversal | ARE THEY CONGRUENT OR SUPPLEMENTARY? | How to set up an equation to solve for x |
|---|---|---|---|
| Corresponding angles | | | |
| Alternate interior angles | | | |
| Alternate exterior angles | | | |
| Same-side (consecutive) interior angles | | | |

| HOW TO SOLVE AN EQUATION IN THE CALCULATOR: | |
|---|---|

## G.2 PROBLEMS:

Lines $a$ and $b$ intersect lines $c$ and $d$.
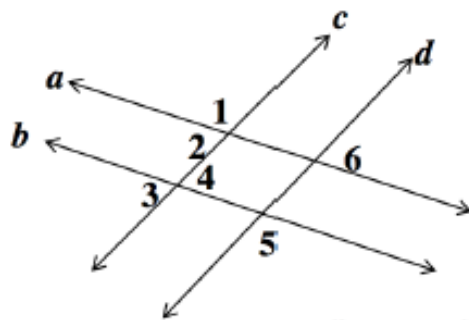


Which statement could be used to prove $a \parallel b$ and $c \parallel d$?

A. $\angle 1$ and $\angle 2$ are supplementary and $\angle 5 \cong \angle 6$

B. $\angle 1 \cong \angle 3$ and $\angle 3 \cong \angle 5$

C. $\angle 3$ and $\angle 5$ are supplementary, and $\angle 5$ and $\angle 6$ are supplementary

D. $\angle 3 \cong \angle 4$ and $\angle 2 \cong \angle 6$

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Lines $a$ and $b$ intersect lines $c$ and $d$.
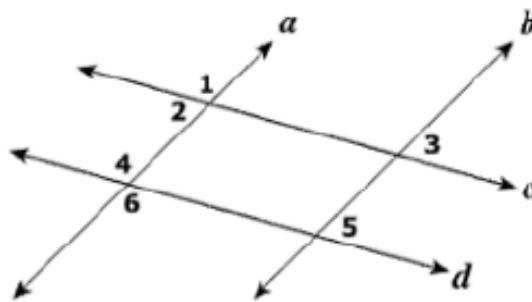


Which of the following statements could be used to prove that $a \parallel b$ and $c \parallel d$ ?

A $\angle 1 \cong \angle 6, \angle 3 \cong \angle 5$

B $\angle 1 \cong \angle 6, \angle 4$ and $\angle 5$ are supplementary

C $\angle 1 \cong \angle 4, \angle 1$ and $\angle 2$ are supplementary

D $\angle 1$ and $\angle 3$ are supplementary, $\angle 1$ and $\angle 6$ are supplementary

**20** Line *c* intersects both line *a* and line *b*. What else must be true for line *c* to be considered the transversal of lines *a* and *b*?

A  Lines *a* and *b* must be parallel.

B  Lines *a* and *b* must intersect.

C  Line *c* must not intersect any other lines.

D  Nothing else is needed.

**Use the figure below for questions 21 and 22 Line *t* intersects lines *a* and *b*.**
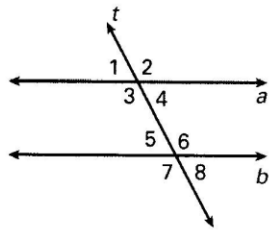


**21** Which angle has to have the same measure as ∠2 for lines *a* and *b* to be parallel?

A  ∠1          C  ∠7

B  ∠3          D  ∠8

**22** Which angle has to be supplementary to ∠4 for lines *a* and *b* to be parallel?

F  ∠6          H  ∠5

G  ∠3          J  ∠8

**23** Line *t* intersects lines *p* and *q*.
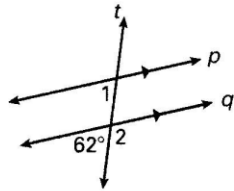


Which statement must be true about ∠1 and ∠2 in order for line *p* and line *q* to be parallel?

A  Their measures must sum to 62°.

B  Their measures must sum to 118°.

C  Their measures must be supplementary.

D  Their measures must be equal.

**24** Line *t* intersects lines *p* and *q*.
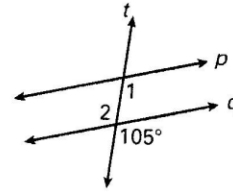


Which statement must be true about ∠1 and ∠2 in order for line *p* and line *q* to be parallel?

F  Their measures must be equal.

G  Their measures must be supplementary.

H  Their measures must sum to 105°.

J  Their measures must sum to 75°.

**Use the figure below for questions 25 and 26 Line *t* intersects lines *a* and *b*.**
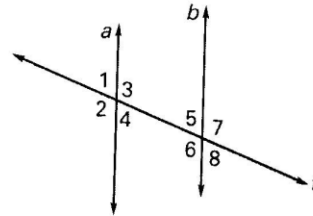


**25** Which angle has to have the same measure as ∠5 for lines *a* and *b* to be parallel?

A  ∠3          C  ∠8

B  ∠1          D  ∠2

**26** Which angle has to be supplementary to ∠3 for lines *a* and *b* to be parallel?

F  ∠6          H  ∠7

G  ∠8          J  ∠4

**27** If line *m* is rotated about its intersection with line *p*, until line *m* is parallel to line *n*, what is the resulting measure of ∠1?
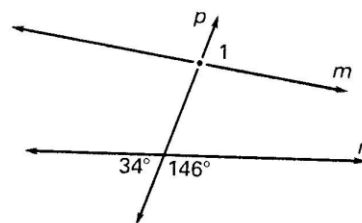


F  34°          H  90°

G  146°          J  112°

**28** Classify the pair ∠1 and ∠8.
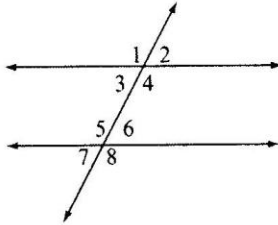


A   Corresponding angles

B   Alternate interior angles

C   Alternate exterior angles

D   Consecutive interior angles

**Use the figure below for questions 29, 30, 31**
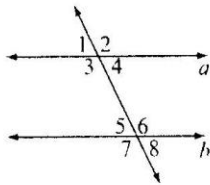


**29** Choose the reason the statement "If $m\angle 3 = 115°$, then $m\angle 5 = 65°$" is true.

A   Alternate Interior Angles Theorem

B   Alternate Exterior Angles Theorem

C   Consecutive Interior Angles Theorem

D   Vertical Angles Theorem

**30** Choose the reason the statement "If $m\angle 1 = 65°$, then $m\angle 5 = 65°$" is true.

F   Alternate Interior Angles Theorem

G   Alternate Exterior Angles Theorem

H   Consecutive Interior Angles Theorem

J   Corresponding Angles Postulate

**31** If $m\angle 6 = 115°$, then $m\angle 3 = $ ?

A   65°

B   115°

C   180°

D   90°

**32** Use the diagram to answer the question below.

$\angle PEA \cong \angle RFE$
**Prove that line PQ is parallel to line RS.**
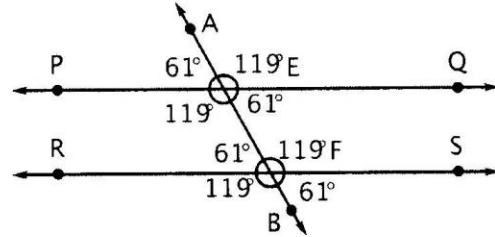


**What reason can be used to prove that lines PQ and RS are parallel?**

F   The distance between $\overleftrightarrow{PQ}$ and $\overleftrightarrow{RS}$ is the same.

G   Corresponding angles are congruent.

H   Supplementary angles are congruent.

J   $\overleftrightarrow{AB}$ is a perpendicular transversal.

**33** Examine the diagram below, where lines m and n are parallel. Which is a valid conclusion and valid reasoning based on the diagram?
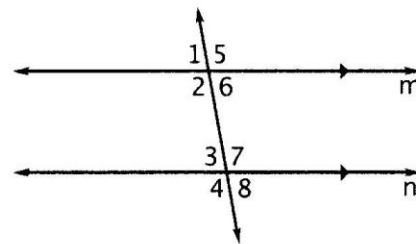


F   ∠2 and ∠8 are congruent because corresponding and vertical angles are congruent.

G   The measures of ∠1 and ∠4 have a sum of 180° because same-side exterior angles are supplementary.

H   ∠5 and ∠3 are congruent because alternate interior angles are congruent.

J   ∠6 and ∠7 are congruent because same-side interior angles are congruent.

Use the figure below for questions 34-35 In the figure, runway 3 crosses runways 1 and 2 and acts as a transversal.
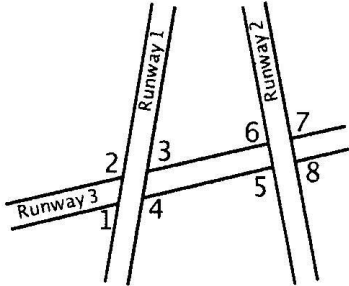


**34** Which pair of angles formed by the runways must be congruent?

A  ∠2 and ∠4

B  ∠1 and ∠2

C  ∠2 and ∠8

D  ∠5 and ∠6

**35** If runways 1 and 2 are to be parallel, what must be true by the Corresponding Angles Postulate?

F  ∠4 ≅ ∠8

G  ∠1 ≅ ∠7

H  ∠2 and ∠7 are supplementary.

J  ∠1 and ∠8 are supplementary.

Use the figure below for questions 36-38 The railroad track represents a transversal to the lines that represent the sides of Pine Street. The sides of the street are parallel.
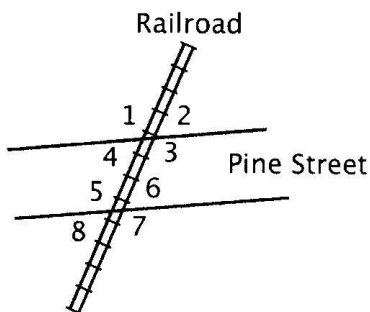


**36** Which statement is justified by the Alternate Exterior Angles Theorem?

A  ∠3 ≅ ∠5

B  ∠2 ≅ ∠8

C  ∠1 and ∠8 are supplementary.

D  ∠4 and ∠5 are supplementary.

**37** Which statement is justified by the Consecutive Interior Angles Theorem?

F  ∠4 ≅ ∠6

G  ∠1 ≅ ∠7

H  ∠3 and ∠6 are supplementary.

J  ∠2 and ∠7 are supplementary.

**38** Which of the following statements is correct?

A  ∠1 and ∠6 are complementary.

B  ∠1 and ∠6 are supplementary.

C  ∠1 and ∠6 are congruent.

D  ∠1 and ∠6 are similar.

**39** A wooden gate has z-shaped boards for support, as shown.
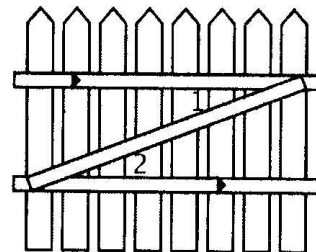


Which of the following statements is true?

F  $m\angle 1 + m\angle 2 = 180$

G  $m\angle 1 + m\angle 2 = 90$

H  $m\angle 2 - m\angle 1 = 2(m\angle 1)$

J  $m\angle 1 + m\angle 2 = 2(m\angle 2)$

## SOL G.3

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation.
 a) investigating and using formulas for finding distance, midpoint, and slope;
 b) applying slope to verify and determine whether lines are parallel or perpendicular;
 c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
 d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

• Find the coordinates of the midpoint of a segment, using the midpoint formula.
• Use a formula to find the slope of a line.
• Compare the slopes to determine whether two lines are parallel, perpendicular, or neither.
• Determine whether a figure has point symmetry, line symmetry, both, or neither.
• Given an image and preimage, identify the transformation that has taken place as a reflection, rotation, dilation, or translation.
• Apply the distance formula to find the length of a line segment when given the coordinates of the endpoints.

## WHAT I NEED TO KNOW:

### FORMULAS

| DISTANCE | MIDPOINT | SLOPE | ENDPOINT |
|---|---|---|---|
|  |  |  |  |

### HOW TO TELL WHETHER LINES ARE PARALLEL OR PERPENDICULAR

| PARALLEL LINES | PERPENDICULAR LINES |
|---|---|
|  |  |

| SYMMETRY | |
|---|---|
| LINE | POINT |
|  |  |
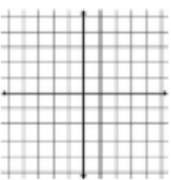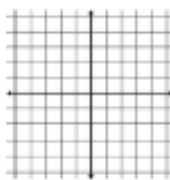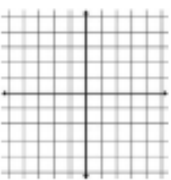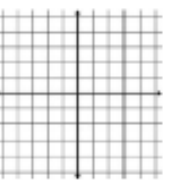
### TRANSFORMATIONS

| TRANSLATION | REFLECTION | ROTATION | DILATION |
|---|---|---|---|
|  |  |  |  |

### WHAT THE FOLLOWING GRAPHS LOOK LIKE

| y = x | y = -x | y = # | x = # |
|---|---|---|---|
|  |  |  |  |

## G.3 PROBLEMS:

Line t contains the points (-4,7) and (5,-8). Plot a point other than point P with integral coordinates that lies on a line that is parallel to t and passes through point P.
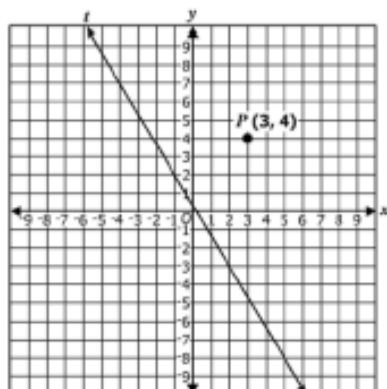
Quadrilateral $QRST$ is to be reflected over the line $y = -x$.

What are the coordinates of point $T'$ after this reflection?
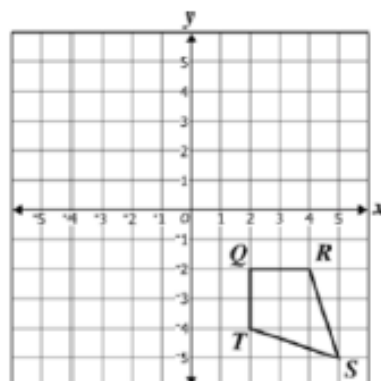
A $(^-4, 2)$

B $(^-2, ^-4)$

C $(2, 4)$

D $(4, ^-2)$

Given: Triangle $ABC$ with vertices located at

$A(1, 1)$, $B(2, -3)$, and $C(-1, -4)$.

Triangle $ABC$ will be reflected over the line $y = x$. What will be the integral coordinates of point $C'$ after this transformation?

$C' (\boxed{\phantom{0}} , \boxed{\phantom{0}} )$

Line $a$ passes through points with coordinates (-4, 5) and (2, -2). What is the slope of a line perpendicular to line $a$?

## Slope of perpendicular line = [            ]

What is the total number of lines of symmetry for this figure?

14

**48** Which lines are perpendicular?

Line 1: $2x + y = 4$

Line 2: $y = x - 7$

Line 3: $\frac{1}{2}x - y = -3$

A  Lines 1 and 2

B  Lines 1 and 3

C  Lines 2 and 3

D  None of the lines are perpendicular.

**49** Which of the following statements are true about lines $w, n, p,$ and $z$?

$w: y = \frac{3}{2}x + 2$

$n: y = \frac{2}{3}x + 6$

$p: y = -\frac{3}{2}x - 3$

$z: y = \frac{2}{3}x + 1$

I. $w \perp p$    II. $n \parallel z$    III. $z \perp p$

F  I only

G  II only

H  III only

J  II and III

**50** Which pair of lines are perpendicular?

A  Line 1: $(8, 12), (7, -5)$
   Line 2: $(-9, 3), (8, 2)$

B  Line 1: $(3, -4), (-1, 4)$
   Line 2: $(2, 7), (5, 1)$

C  Line 1: $(-3, 1), (-7, -2)$
   Line 2: $(2, -1), (8, 4)$

D  Line 1: $(-1, 3), (4, 1)$
   Line 2: $(-2, -1), (3, -3)$

**51** Grant wrote an equation of a line through the point $(4, 1)$ that is perpendicular to the one shown. What other point lies on his line?
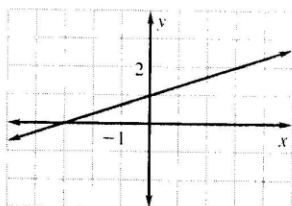


F  $(2, 7)$    H  $(-1, 12)$

G  $(3, 3)$    J  $(4, 5)$

**52** If two different lines with equations $y = m_1x + b_1$ and $y = m_2x + b_2$ are parallel, which of the following must be true?

F  $b_1 = b_2$ and $m_1 \neq m_2$

G  $b_1 \neq b_2$ and $m_1 = m_2$

H  $b_1 \neq b_2$ and $m_1 \neq m_2$

J  $b_1 = b_2$ and $m_1 = m_2$

**53** Which equation is an equation of the line parallel to $3x + 4y = 7$ that passes through the point $(2, -1)$?

A  $y = \frac{4}{3}x - \frac{11}{3}$

B  $y = -\frac{4}{3}x + \frac{5}{3}$

C  $y = \frac{3}{4}x - \frac{5}{2}$

D  $y = -\frac{3}{4}x + \frac{1}{2}$

**54** Figure $ABCD$ is a parallelogram. Which statement would prove that parallelogram $ABCD$ is a rhombus?
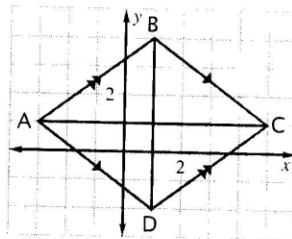


F  $AB = CD$

G  $AC = BD$

H  slope $\overline{AB} =$ slope $\overline{CD}$

J  $(\text{slope } \overline{AC})(\text{slope } \overline{BD}) = -1$

**55** Which lines are parallel?
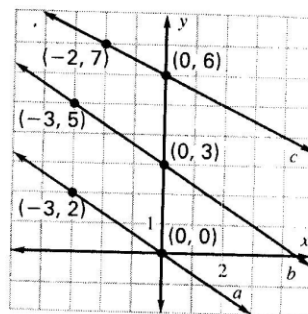


A  $a \parallel c$    C  $a \parallel b$

B  $b \parallel c$    D  $a \parallel b \parallel c$

**56** Which type of symmetry does the figure possess?
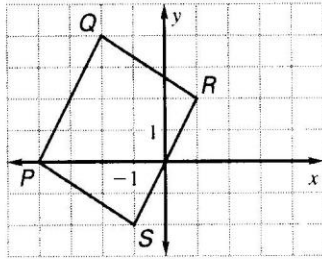


A  180° clockwise rotation about the point $\left(\frac{-3}{2}, 1\right)$.

B  90° clockwise rotation about the point $\left(\frac{-3}{2}, 1\right)$.

C  Reflection across a line through points $Q$ and $S$.

D  Reflection across a line through points $P$ and $R$.

**57** A regular pentagon has many symmetries. Two of them are _____ and _____ rotation symmetry.

F  72° clockwise; 144° counterclockwise

G  36° clockwise; 108° counterclockwise

H  90° clockwise; 270° counterclockwise

J  60° clockwise; 240° counterclockwise

**58** In the figure below, which segment represents a 90° clockwise rotation of segment $AB$ about $P$?
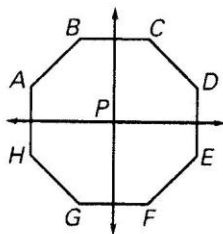


F  $\overline{BC}$

G  $\overline{EF}$

H  $\overline{HG}$

J  $\overline{CD}$

**59** The design below is made of congruent isosceles trapezoids
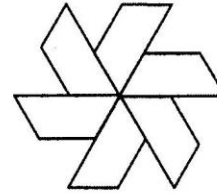


Each of the trapezoids has _____ symmetry, and the figure as a whole has _____ symmetry. Choose the *best* pair of answers to fill in the blanks.

A  Reflection; reflection

B  Reflection; rotation

C  Rotation; reflection

D  Rotation; rotation

**60** Which of the following *best* describes the line of reflection symmetry in the figure below?
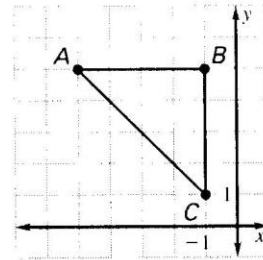


F  Vertical line through the point $(-2, 3)$

G  Horizontal line through the point $(-2, 3)$

H  Diagonal line through point $B$

J  Diagonal line through points $A$ and $C$

**61** The shortest distance between point $W$ and a line of reflection is 64 inches. What will be the distance between $W$ and its image $W'$?

F  8 in.

G  32 in.

H  64 in.

J  128 in.

**62** Emily wants to transform △*JKL* so that △*J'K'L'* has the coordinates *J'*(−3, 5), *K'*(0, 4), and *L'*(−5, 1).
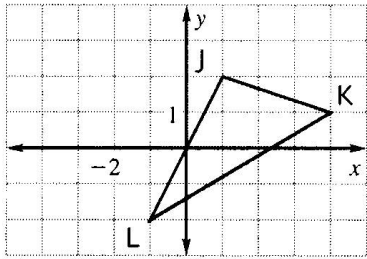


Which transformation should she perform?

**A** Translate △*JKL* 4 units left and 3 units up.

**B** Rotate △*JKL* counterclockwise 90°.

**C** Reflect △*JKL* across the *x*-axis.

**D** Reflect △*JKL* across the *y*-axis.

**63** Lauren is working on a problem where she must rotate point *J* 90° clockwise about the origin. If the coordinates of *J* are (2, −7), where should she plot the final image?

**F** (2, 7)

**G** (−2, 7)

**H** (7, 2)

**J** (−7, −2)

**64** In quadrilateral *RSTU*, the coordinates of *R* are (3, −5). What are the coordinates of the image of *R* after a rotation of 180° counterclockwise?

**A** *R'*(−3, −5)

**B** *R'*(−5, −3)

**C** *R'*(5, 3)

**D** *R'*(−3, 5)

**65** If quadrilateral *EFGH* was reflected over the *x*-axis to form *E'F'G'H'*, what would be the coordinates of *E'*?



**F** (5, −7)

**G** (5, 7)
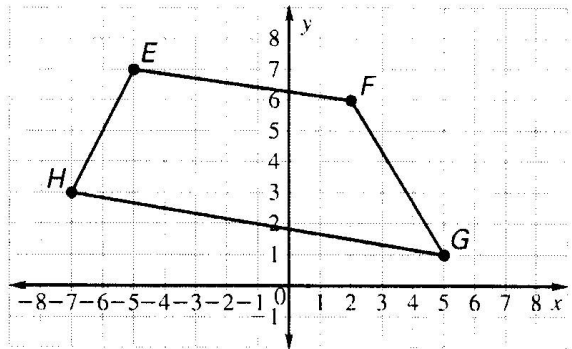
**H** (−5, 7)

**J** (−5, −7)

**66** One vertex of a triangle is located at the point *P*(−2, 7). If a scale factor of 2.2 is used for a dilation of the triangle, where will the image point *P'* be located?

**A** (−4.4, 15.4)

**B** (−4.2, 4.8)

**C** (0.2, 9.2)

**D** (−0.9, 3.2)

**67** △*ABC* is to be reflected across the *y*-axis. What will be the coordinates of *B'*?
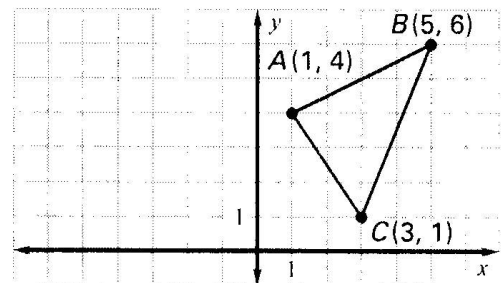


**F** (5, −6)

**G** (−1, 4)

**H** (−5, 6)

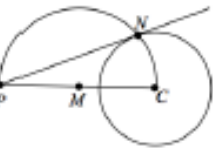**J** (3, −1)

| SOL G.4 |
| --- |

The student will construct and justify the constructions of
a) a line segment congruent to a given line segment;
b) the perpendicular bisector of a line segment;
c) a perpendicular to a given line from a point not on the line;
d) a perpendicular to a given line at a given point on the line;
e) the bisector of a given angle,
f) an angle congruent to a given angle; and
g) a line parallel to a given line through a point not on the given line.

- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- Construct the inscribed and circumscribed circles of a triangle.
- Construct a tangent line from a point outside a given circle to the circle.

## WHAT I NEED TO KNOW:

| WHAT EACH CONSTRUCTION LOOKS LIKE, THE STEPS, AND THE JUSTIFICATIONS | | |
| --- | --- | --- |
| CONSTRUCTION | STEPS | JUSTIFICATION |
|  | | |
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| CONSTRUCTION | STEPS | JUSTIFICATION |
|---|---|---|
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## SOL G.4

The student will construct and justify the constructions of
    a) a line segment congruent to a given line segment;
    b) the perpendicular bisector of a line segment;
    c) a perpendicular to a given line from a point not on the line;
    d) a perpendicular to a given line at a given point on the line;
    e) the bisector of a given angle,
    f) an angle congruent to a given angle; and
    g) a line parallel to a given line through a point not on the given line.

- Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
- Construct the inscribed and circumscribed circles of a triangle.
- Construct a tangent line from a point outside a given circle to the circle.

## WHAT I NEED TO KNOW:



**Construction 1** — Segment Congruent To Given Segment: Start, Step 1, Step 2, Step 3, Finish

**Construction 2** — Angle Congruent To Given Angle: Start, Step 1, Step 2, Step 3, Finish

**Construction 3** — Angle Bisector: Start, Step 1, Step 2, Step 3, Finish

**Construction 4** — Perpendicular Bisector of Line Segment: Start, Step 1, Step 2, Finish

| **Construction 5** | Start | Step 1 | Step 2 | Step 3 | Finish |
|---|---|---|---|---|---|
| ***Perpendicular to Line from Point on Line*** | | | | | |

| **Construction 6** | Start | Step 1 | Step 2 | Step 3 | Finish |
|---|---|---|---|---|---|
| ***Perpendicular to Line from Point Not on Line*** | | | | | |

***Parallel to line through point not on the line***

**Construction 7**

Start

Step 1

Step 2

Step 3

Step 4

Step 5

$\angle 1 = \angle 2 \rightarrow h \parallel k$

Postulate 11

**Justification**

## G.4 PROBLEMS:

Which construction represents a correct first step in constructing a line segment perpendicular to $\overline{JK}$ through point $P$?
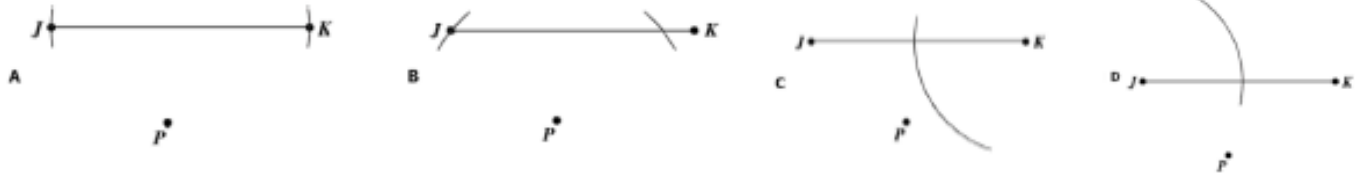


Ben plans to bisect $\angle ABC$ to create the congruent angles $ABD$ and $CBD$.
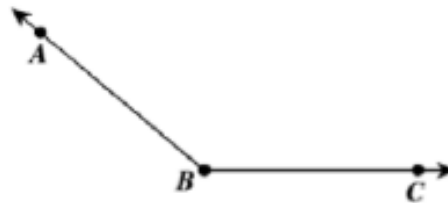


Which angle is congruent to $\angle ABD$ and $\angle CBD$?

○ A

○ C

○ B

○ D
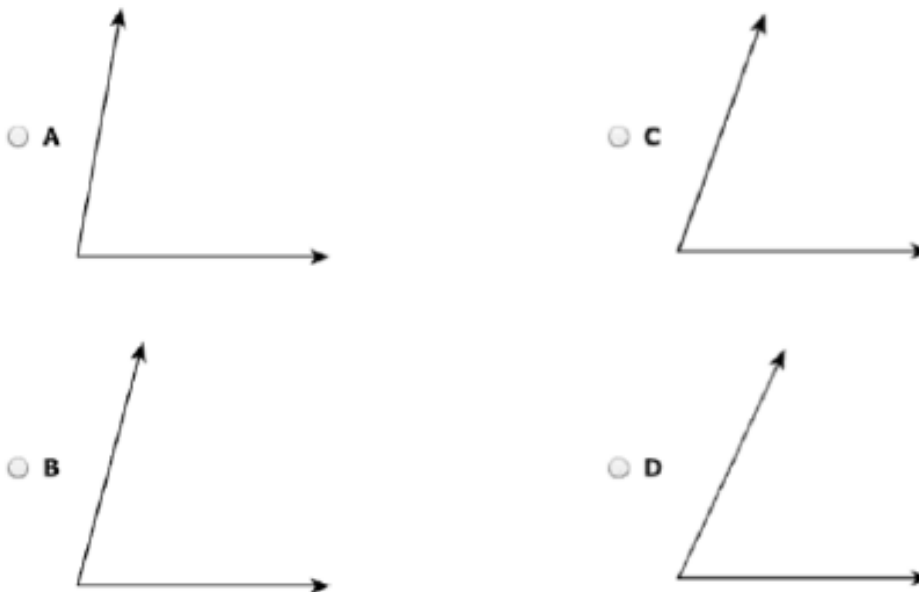


Point $A$ represents a vertex of an equilateral triangle inscribed in circle $O$.
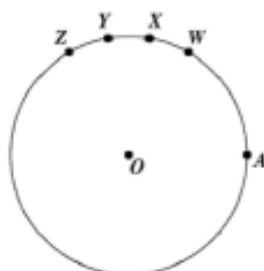


A Point $W$

B Point $X$

C Point $Y$

D Point $Z$

Which other point is also a vertex of this equilateral triangle?

22

**68** In constructing a line segment congruent to a given line segment, what is the purpose of drawing an arc that intersects the second segment?

**A** To set the compass at the length of the original segment

**B** To mark the end of the second segment

**C** To establish the angle between the two segments

**D** To ensure that the two segments are parallel

**69** The figure shows the construction of a line segment congruent to a given line segment. What is the distance from point $C$ to point $P$?
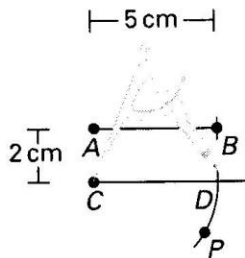


**F** 2 cm

**G** 3 cm

**H** 5 cm

**J** 7 cm

**70** In constructing a line segment $\overline{CD}$ congruent to a given line segment $\overline{AB}$. what is the purpose of placing the compass at $C$?

**A** Preparing to measure the distance from point $C$ to point $A$

**B** Preparing to set the compass to the length of $\overline{AB}$

**C** Preparing to connect points $C$ and $D$

**D** Preparing to mark point $D$

**71** Some of the steps for constructing a line segment $\overline{CD}$ congruent to a given line segment $\overline{AB}$ are listed below. Which of these is the *first* step?

**F** Place the compass at point $C$.

**G** Mark point $D$ on the new segment.

**H** Label one end of the new segment as point $C$.

**J** Draw a segment longer than $\overline{AB}$.

**72** For which of the following would be it be useful to know how to construct a line segment congruent to a given line segment? Choose the *best* answer.

**A** To draw a perfect circle

**B** To draw a quadrilateral with two congruent sides

**C** To draw a trapezoid with two perfectly parallel sides

**D** To draw a perfect rectangle

**73** What is being done in the figure, as part of constructing a line segment congruent to a given line segment?
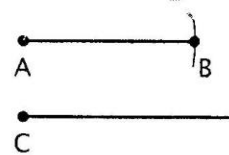


**F** The location of point $A$ is being determined.

**G** The location of point $B$ is being determined.

**H** The compass is being set to the length of segment $\overline{AB}$.

**J** Segment $\overline{AB}$ is being drawn.

For questions 74; 75 use the figure below. It shows the construction of a perpendicular to given line $n$ through given point $P$. All arcs were drawn with the same compass setting.
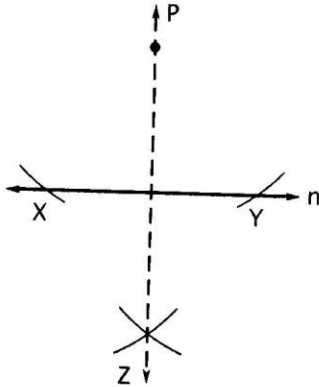


**74** Suppose that distance $PY$ is 6.2 cm. What is distance $XZ$?

A  12.4 cm

B  8.8 cm

C  6.2 cm

D  Cannot be determined

**75** Suppose that distance $XZ$ is 4.9 cm. What is distance $PZ$?

F  9.8 cm

G  6.9 cm

H  4.9 cm

J  Cannot be determined

figure shows the construction of the bisector of $\angle CAB$.



Step 1   Step 2   Step 3
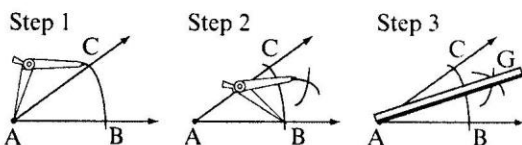
**76** Which distances can you assume are equal?

A  $AC$ and $AG$

B  $AC$ and $AB$

C  $AG$ and $AB$

D  $AC$ and $BC$

The figure shows an attempt to construct a perpendicular to line $n$ from point $P$. Distances $PX$ and $PY$ are equal and distances $XZ$ and $YZ$ are equal, but $PX$ does not equal $XZ$.
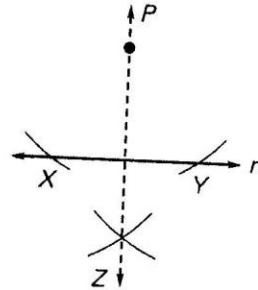


**77** What is wrong with this construction?

F  Line $\overleftrightarrow{PZ}$ will not be perpendicular to line $n$.

G  Line $\overleftrightarrow{PZ}$ will not bisect segment $\overline{XY}$.

H  The perpendicular will not pass exactly through point $P$.

J  Nothing is wrong. The construction will still work.

The figure shows the construction of a perpendicular to given line $n$ through given point $P$.
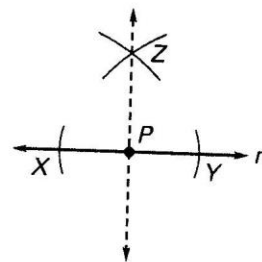


**78** Suppose that distance $XP$ is 7.3 cm. What is distance $YZ$?

A  16.3 cm

B  14.6 cm

C  7.3 cm

D  Cannot be determined

**79** **What went wrong in the construction below?**
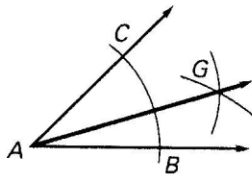


F   Arc $\overparen{BC}$ is not centered at $A$.

G   Distance $AG$ should not be longer than distances $AC$ and $AB$.

H   The two arcs through $G$ have different radii.

J   Nothing is wrong; the construction is done correctly.

**80** **George is constructing a line parallel to line $PQ$ that passes through point $R$. Which of the following should be his first step?**

F



G



H



J



**81** **What went wrong with the following attempt to copy $\angle CAB$?**
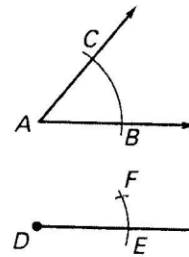


A   The reference point for the compass should be $D$, not $E$.

B   Point $F$ should be drawn on the original angle, not the copy.

C   The compass being used to draw point $F$ is not set at distance $BC$.

D   Angle $CAB$ was drawn wider than it should be.

**82** **The figure shows the construction of a line through point $P$ parallel to line $m$.**
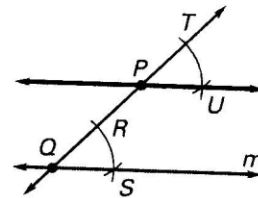


If $m\angle RQS = 50°$, what is $m\angle TPU$?

F   50°           H   80°

G   65°           J   130°
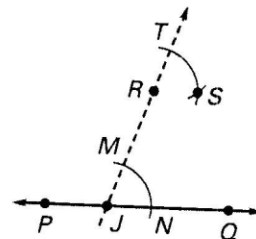


**83** **The figure shows point $S$ being drawn to meet certain conditions. What are the conditions?**

A   $RS = JN$ and $RN = SJ$

B   $TS = MN$ and $SP = MQ$

C   $RS = JN$ and $TS = MN$

D   $RJ = SN$ and $SP = MQ$

## SOL G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will
   a) order the sides by length, given the angle measures;
   b) order the angles by degree measure, given the side lengths;
   c) determine whether a triangle exists; and
   d) determine the range in which the length of the third side must lie.
   These concepts will be considered in the context of real-world situations.

• Order the sides of a triangle by their lengths when given the measures of the angles.
• Order the angles of a triangle by their measures when given the lengths of the sides.
• Given the lengths of three segments, determine whether a triangle could be formed.
• Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie.
• Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles.

## WHAT I NEED TO KNOW:

### HOW TO ORDER THE SIDES OF A TRIANGLE BY THEIR LENGTHS GIVEN THE MEASURES OF THE ANGLES

### HOW TO ORDER THE ANGLES OF A TRIANGLE BY THEIR MEASURES WHEN GIVEN THE LENGTHS OF THE SIDES

### HOW TO DETERMINE IF 3 GIVEN LENGTHS FORM A TRIANGLE

### HOW TO DETERMINE THE RANGE OF THE THIRD SIDE OF THE TRIANGLE GIVEN 2 SIDE LENGTHS

## G.5 PROBLEMS:

**Given:** Triangle $ABC$ with $AB$ = 42 and $BC$ = 20

**Which of the following are possible lengths for $AC$?**

12    20    22    32    42    50    62    70

---

Given the following diagram of a triangle, write in the angle measures and side lengths from the given box that would make the triangle possible. (Figure not drawn to scale.)

| 100° | 80° | 60° | 4 |
|------|-----|-----|---|
| 1    | 8   | 40° | 3 |

40°

7

---
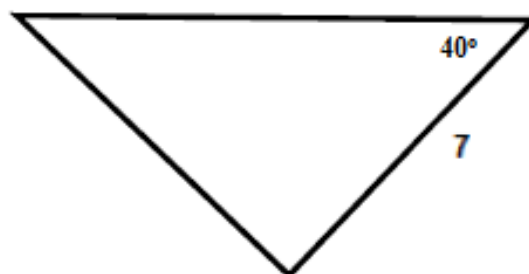
Two sides of a triangle measure 9 inches and 13 inches. Write the numbers in the boxes that would correctly represent the range of the third side of the triangle.

☐ $< X <$ ☐

| 9 | 13 | -4 | 11 |
|---|----|----|----|
| 4 | 21 | 5  | 22 |

---

The diagram is a map showing Jaime's house, Kay's house and the grocery store. Write the segments that represent the distances from each place in order from least to greatest.

Store (S)

$(2y + 6)°$

$(8y + 10)°$    $(y + 32)°$

Jaime's house (J)        Kay's house (K)
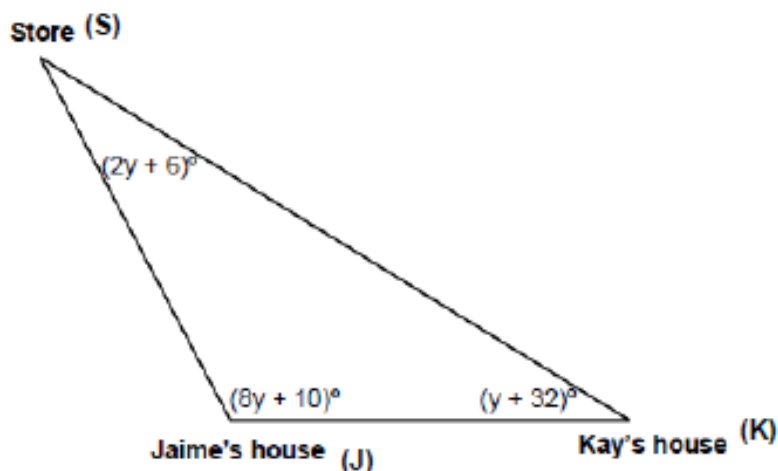
## SOL G.6

The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs.

• Use definitions, postulates, and theorems to prove triangles congruent.
• Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are congruent.
• Use algebraic methods to prove two triangles are congruent.

## WHAT I NEED TO KNOW:

| 5 METHODS OF PROVING TRIANGLES ARE CONGRUENT | |
|---|---|
|  | |
|  | |
|  | |
|  | |
|  | |

| 2 METHODS THAT DO NOT PROVE TRIANGLES ARE CONGRUENT | |
|---|---|
|  | |
|  | |

| PROPERTIES THAT HELP PROVE TRIANGLES ARE CONGRUENT |
|---|
| |

| FORMULA USED WHEN PROVING TRIANGLES ARE CONGRUENT USING COORDINATE GEOMETRY |
|---|
| |

| WHAT DOES CPCTC MEAN? | HOW TO USE AND WRITE A ≅ STATEMENT |
|---|---|
| | |

## SOL G.7

• Use definitions, postulates, and theorems to prove triangles similar.
• Use algebraic methods to prove that triangles are similar.
• Use coordinate methods, such as the distance formula, to prove two triangles are similar.

## WHAT I NEED TO KNOW:

### 3 METHODS OF PROVING TRIANGLES ARE SIMILAR



| | |
|---|---|
| (diagrams) | |
| (diagrams) | |
| (diagrams) | |

### CORRESPONDING SIDES ARE PROPORTIONAL AND CORRESPONDING ANGLES ARE CONGRUENT

### WHAT DOES THIS MEAN?

| HOW TO SET UP A PROPORTION WHEN SOLVING FOR A MISSING SIDE | WHAT IS A SIMILARITY RATIO (SCALE FACTOR) AND HOW TO USE IT |
|---|---|
| | |

## G.7 PROBLEMS:

**For what value of $x$ is $\triangle ABC \sim \triangle DEF$ ?**
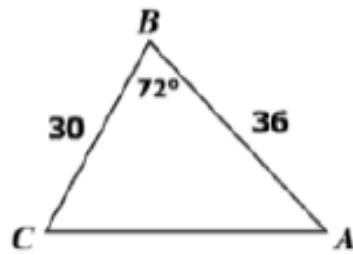


A  18

B  21

C  25

D  72

---

Complete the proof.



Given: $\overline{BA} \perp \overline{AC}$
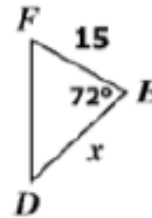$\overline{DC} \perp \overline{AC}$

Prove: $\triangle BFA \sim \triangle CFD$

| Statements | Reasons |
|---|---|
| 1. Given: $\overline{BA} \perp \overline{AC}$ $\overline{DC} \perp \overline{AC}$ | 1. Given |
| 2. $\overline{BA} \parallel \overline{DC}$ | 2. If two lines are perpendicular to a third line, then the two lines are parallel. |
| 3. | 3. If two parallel lines are cut by a transversal, alternate interior angles are congruent. |
| 4. $\triangle BFA \sim \triangle CFD$ | 4. |

| |
|---|
| $\angle DFC \cong \angle BFA$; $\angle DAB \cong \angle BCD$ |
| $\angle CBA \cong \angle ADC$; $\angle BAD \cong \angle DCB$ |
| $\angle CDA \cong \angle BAD$; $\angle CBA \cong \angle BCD$ |
| Angle-Angle (AA) Postulate |
| Side-Angle-Side (SAS) Postulate |

---

Given: $\triangle ACF$ is subdivided into smaller triangles
$\overline{AC} \perp \overline{AF}$ and $\overline{AC} \perp \overline{BE}$ and $\overline{AE} \perp \overline{CF}$
Point $B$ lies on $\overline{AC}$ and points $D$ and $E$ lie on $\overline{CF}$

Based on the given information, identify two triangles that may NOT be similar.
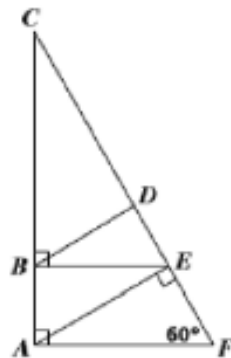


| $\triangle ACF$ | $\triangle BCE$ | $\triangle BEA$ | $\triangle DBE$ | $\triangle EAF$ |
|---|---|---|---|---|

Given: $\overline{AB} \parallel \overline{CD}$

Prove: $\triangle ABF \sim \triangle DCF$
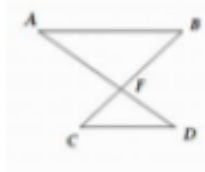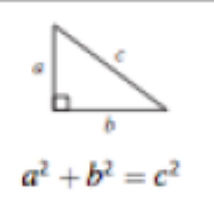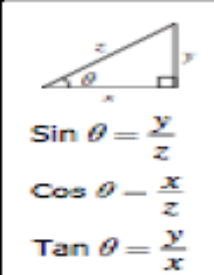
## SOL G.8

The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

• Determine whether a triangle formed with three given lengths is a right triangle.
• Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles.
• Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles.
• Solve problems involving right triangles, using sine, cosine, and tangent ratios.
• Solve real-world problems, using right triangle trigonometry and properties of right triangles.
• Explain and use the relationship between the sine and cosine of complementary angles.

| CALCULATOR MODE | WHAT I NEED TO KNOW: |
|---|---|

## WHAT FORMULAS TO USE FROM THE FORMULA SHEET AND WHEN TO USE THEM

| | WHEN TO: | | | WHEN TO: | | |
|---|---|---|---|---|---|---|
| $a^2 + b^2 = c^2$ | ADD | SUBTRACT | $Sin\ \theta = \frac{y}{z}$ $Cos\ \theta = \frac{x}{z}$ $Tan\ \theta = \frac{y}{x}$ | MULTIPLY | DIVIDE | USE INVERSE |
| | | | | | | |

## HOW TO DETERMINE WHETHER A TRIANGLE FORMED WITH THREE GIVEN LENGTHS IS A RIGHT TRIANGLE

| |
|---|
| |

## SPECIAL RIGHT TRIANGLES

| $45° - 45° - 90°$ | $30° - 60° - 90°$ |
|---|---|
| | |

| STEPS TO SOLVING FOR MISSING SIDE OR ANGLE USING TRIG. RATIOS | HOW TO USE AND EXPLAIN THE RELATIONSHIP BETWEEN THE SINE AND COSINE OF COMPLEMENTARY ANGLES |
|---|---|
| STEP 1: | |
| STEP 2: | |
| STEP 3: | |
| STEP 4: | |

## G.8 PROBLEMS:

A ladder leans against a wall. The bottom of the ladder is 10 feet from the base of the wall, and the top of the ladder makes an angle of 25° with the wall. Find the length, $x$, of the ladder.
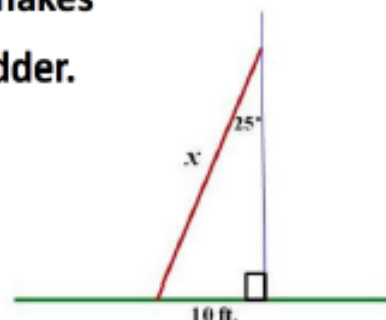


The figure represents the side view of a rectangular frame for metal shelves. Two diagonal braces support the frame.



8 ft

2 ft
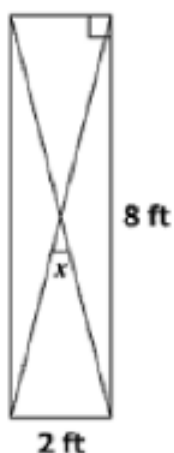
This figure models a gate that has been constructed using two parallel vertical boards with a diagonal board connecting them. Identify all of the statements that must be true.
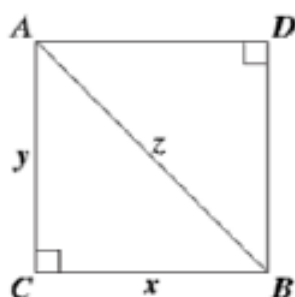


| $\sin\angle CAB + \cos\angle CAB = 180°$ |
| --- |
| $\sin\angle CAB = \cos\angle CBA$ |
| $\angle CAB \cong \angle DAB$ |
| $x^2 + y^2 = z^2$ |
| $\overline{AD} \| \overline{CB}$ |

**84** Which is the shortest side of triangle *ABC*?
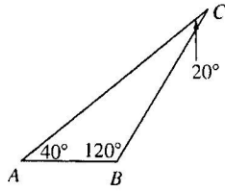


**A** $\overline{AB}$

**B** $\overline{BC}$

**C** $\overline{AC}$

**D** Cannot be determined

**85** In triangle *XYZ*, $m\angle X = 39°$, $m\angle Y = 82°$, and $m\angle Z = 59°$. Which of the following lists the sides of $\triangle XYZ$ from smallest to largest?

**F** $\overline{XY}, \overline{XZ}, \overline{YZ}$

**G** $\overline{YZ}, \overline{XY}, \overline{XZ}$

**H** $\overline{XZ}, \overline{YZ}, \overline{XY}$

**J** $\overline{XZ}, \overline{XY}, \overline{YZ}$

**86** A new triangular walking track is being constructed. One side is 30 meters long and another side is 45 meters long. If *P* represents the perimeter of the track, which inequality gives the range of values for *P*?

**A** $15\,m < P < 45\,m$

**B** $15\,m < P < 75\,m$

**C** $30\,m < P < 45\,m$

**D** $30\,m < P < 75\,m$

**87** Two sides of a triangle have sides 5 and 20. The length of the third side must be greater than _____ and less than ____.

**F** 5, 20

**G** 15, 25

**H** 14, 26

**J** 4, 21

**88** Triangle *ABC* is shown below. Which of the following inequalities is true?
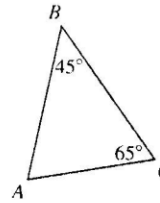


**A** $AC > AB$

**B** $BC > AB$

**C** $AB > BC > AC$

**D** $AB > AC > BC$

In the figure below, a cable supports a radio tower. The tower is 385 feet tall. What is the approximate distance from the anchor point of the cable to the base of the tower?
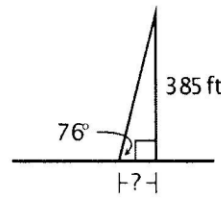


**89**

**A** 93.1 ft

**B** 96.0 ft

**C** 373.6 ft

**D** 396.8 ft

**90** Which set can represent the side lengths of a right triangle?

**F** 5, 11, 12

**G** $2, 4, 2\sqrt{5}$

**H** $6, 7, 2\sqrt{21}$

**J** $5, 7, 5\sqrt{3}$

**91** Which set of lengths can represent the sides of a right triangle?

**F** 5, 12, 13

**G** 7, 8, 10

**H** 6, 6, 9

**J** 8, 12, 14

**92** Melanie is planting a triangular shaped garden. Her landscape designer told her to plant roses along the longest edge of the garden. Along which edge should Melanie plant roses?

M

59°

L        N

**F** $\overline{LM}$          **H** $\overline{LN}$

**G** $\overline{MN}$          **J** Both $\overline{LM}$ and $\overline{MN}$

**93** A surveyor made the following diagram showing his measurements using three landmarks at points $A$, $B$, and $C$. The surveyor measured the lengths of the three sides as 119.0 mete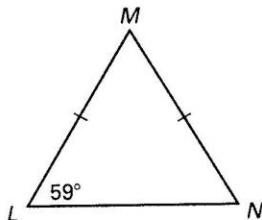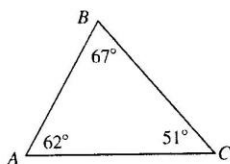rs, 141.0 meters, and 135.2 meters, but he forgot to write down which side each measurement belongs to. Which of the following is correct?
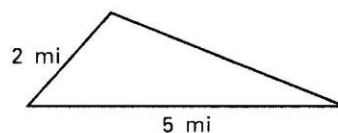
B

67°

62°        51°

A              C

**A** $AB = 141.0$, m, $BC = 119.0$ m, $AC = 135.2$ m

**B** $AB = 135.2$ m, $BC = 119.0$ m, $AC = 141.0$ m

**C** $AB = 119.0$ m, $BC = 135.2$ m, $AC = 141.0$ m

**D** $AB = 135.2$ m, $BC = 141.0$ m, $AC = 119.0$ m

**94** Triangle $ABC$ has $AB = 13$, $BC = 9$, and $AC = 10$. Which of the following inequalities is correct?

**A** $m\angle A > m\angle B > m\angle C$

**B** $m\angle B > m\angle C > m\angle A$

**C** $m\angle C > m\angle B > m\angle A$

**D** $m\angle A > m\angle C > m\angle B$

**95** The figure shows the route Daniel took while riding his bicycle after school.

2 mi

5 mi

Which of the following is *not* a possible measure for the third side of the triangle?

**F** 4 mi

**G** 5 mi

**H** 6 mi

**J** 7 mi

**96** Which group of side lengths can be used to construct a triangle?

**A** 3 yd, 4 ft, 5 yd

**B** 3 yd, 5 ft, 8 ft

**C** 11 in., 16 in., 27 in.

**D** 2 ft, 11 in., 12 in.

**97** The figure shows the outline of a flower garden.

8 ft        12 ft
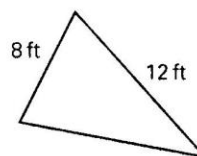
Which of the following is a possible measure for the third side of the garden?

**F** 4 ft          **H** 20 ft

**G** 8 ft          **J** 24 ft

**98** Which of the following inequalities is correct?

$(6x - 14)°$

$p$          $q$

$(5x + 8)°$          $(3x + 4)°$

$t$
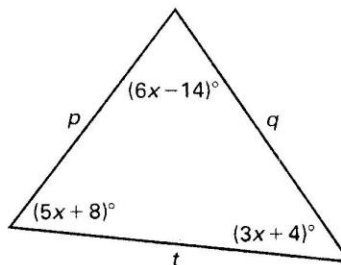
**A** $q > t > p$          **C** $t > p > q$

**B** $p > t > q$          **D** $t > q > p$

**34**

**99** What is the perimeter of quadrilateral *RSTU* shown below?
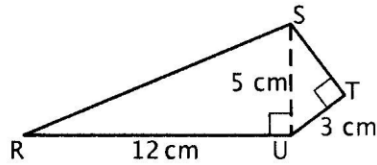


| | | | |
|---|---|---|---|
| **F** | 20 cm | **H** | 32 cm |
| **G** | 30 cm | **J** | 37 cm |

**100** Wires are used to stabilize a telephone pole that is 50 feet high. A wire from the top of the pole to the ground is 63 feet long. To the nearest tenth of a foot, how far from the bottom of the pole is the wire anchored in the ground?
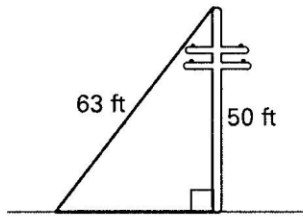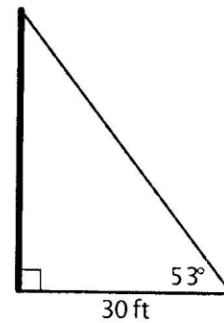


| | | | |
|---|---|---|---|
| **A** | 13.0 ft | **C** | 40.2 ft |
| **B** | 38.3 ft | **D** | 80.4 ft |

**101** The figure shows a ramp leading up to a loading dock that forms a 15° angle with the ground. The loading dock height is 4 feet. What is the approximate distance from point *A* to point *B* to the nearest hundredth foot?
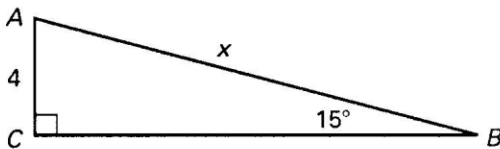


| | | | |
|---|---|---|---|
| **F** | 4.14 ft | **H** | 15.45 ft |
| **G** | 14.93 ft | **J** | 26.67 ft |

**102** To the nearest tenth of a foot, what is the height of the telephone pole in the diagram?



| | | | |
|---|---|---|---|
| **A** | 24.0 ft | **C** | 39.8 ft |
| **B** | 30.6 ft | **D** | 53.0 ft |

**103** About how far is the lighthouse from the boat?
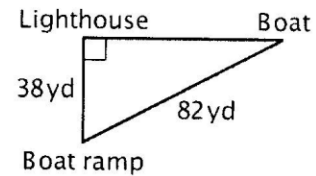


| | | | |
|---|---|---|---|
| **F** | 6.6 yd | **H** | 72.7 yd |
| **G** | 11 yd | **J** | 90.4 yd |

**104** Triangle *DEF* is a right triangle.
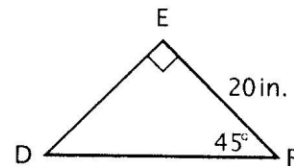


What is the length of $\overline{DF}$?

**A** $20\sqrt{3}$ in.

**B** 40 in.

**C** $10\sqrt{2}$ in.

**D** $20\sqrt{2}$ in.

**105** What is the value of $y$ in the triangle below?



**A** 10

**B** $10\sqrt{2}$

**C** $5\sqrt{3}$

**D** $\dfrac{5\sqrt{3}}{3}$

**106** A waterski jump has the measurements shown in the figure below. What ratio could be used to find the measure of $\angle C$, the angle of inclination for the ramp?



**F** $\cos C = \dfrac{5.5}{20.5}$

**G** $\cos C = \dfrac{5.5}{21.2}$

**H** $\sin C = \dfrac{5.5}{20.5}$

**J** $\sin C = \dfrac{5.5}{21.2}$

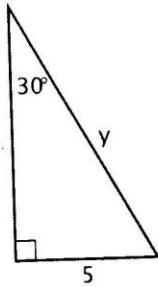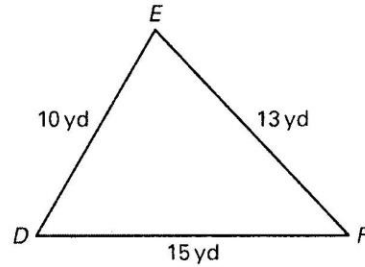**107** A surveyor has measured the distances from a point $P$ to the east and west ends of a pond, as shown in the figure below. What is the distance across the pond from east to west?
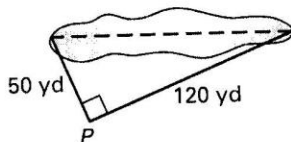


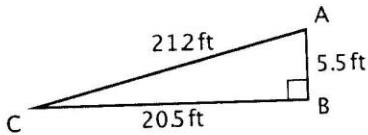**A** 150 yd

**B** 140 yd

**C** 130 yd

**D** 120 yd

**108** A farmer has three sections of fence that he is using to make an enclosure for a goat. Triangle *DEF* represents the enclosure. The farmer wants to put the goat's water in the widest corner of the enclosure. Which angle represents this corner?



**F** $\angle D$      **H** $\angle F$

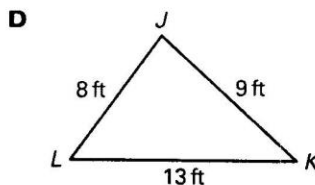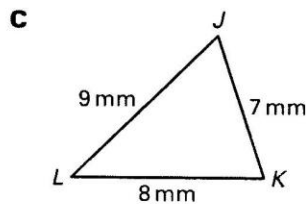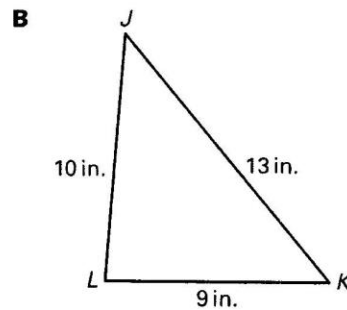**G** $\angle E$      **J** Cannot be determined

**109** In which of the following triangles is $\angle K$ the smallest angle?

**A**



**B**



**C**



**D**

**110** As part of a large art mural, Michael is making a tessellation using tiles in the shape of △RST shown below. On each tile, Michael needs to glue a pebble to the corner with the largest angle. Which angle represents this corner?
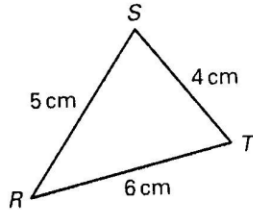


**F**  ∠R

**G**  ∠S

**H**  ∠T

**J**  Cannot be determined

**111** Triangle *GHI* has a perimeter of 42 inches. *GH* = 15 inches and *HI* = 11 inches. Which of the following lists the angles of △*GHI* from smallest to largest?

**A**  ∠I, ∠G, ∠H

**B**  ∠G, ∠I, ∠H

**C**  ∠I, ∠H, ∠G

**D**  ∠H, ∠G, ∠I

**112** Triangle *ABC* has a perimeter of 100. *AB* = 2x + 7, *BC* = 3x − 1, and *AC* = 4x − 5. Which of the following inequalities is correct?

**F**  $m\angle B > m\angle A > m\angle C$

**G**  $m\angle A < m\angle B < m\angle C$

**H**  $m\angle A > m\angle C > m\angle B$

**J**  $m\angle C > m\angle B > m\angle A$

**113** Which of the following sets of numbers could *not* represent the lengths of the sides of a triangle?

**A**  3, 9, 14

**B**  6, 8, 12

**C**  15, 20, 30

**D**  11, 11, 19

**114** Tong is making a triangular shaped frame out of three strips of wood. One of the strips is 10 centimeters long and a second is 15 centimeters long. What could be the length of the third strip?

**A**  4 cm

**B**  5 cm

**C**  20 cm

**D**  26 cm

**115** A tile company is making a new size of triangular shaped tiles. What could be the dimensions of this new size?

**F**  41 mm, 21 mm, 20 mm

**G**  16 mm, 40 mm, 10 mm

**H**  41 mm, 5 mm, 49 mm

**J**  29 mm, 25 mm, 50 mm

**116** Two sides of a triangle measure 14 and 9. Which of the following cannot be the perimeter of the triangle?

**A**  28

**B**  37

**C**  42

**D**  46

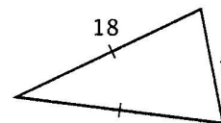**117** A triangle has one side of length 20 and another of length 10. Which of the following best describes the possible lengths of the third side?

**A**  $10 < x < 20$

**B**  $11 < x < 29$

**C**  $20 < x < 30$

**D**  $10 < x < 30$

**118** The triangle below is isosceles.



If *t* is a whole number, what is its largest possible value?

**F**  35          **H**  37

**G**  36          **J**  38

**19** Maxwell is cutting out a triangular piece of stained glass to make a window. One side of the triangle is 9 inches long. If the length of the second side must be between 4 and 14 inches, what is the length of the third side?

A  4 in.

B  5 in.

C  9 in.

D  14 in.

**20** A sign company is making a large triangle on a sign out of reflective tape. The triangle is shown below. If $x$ is the total number of feet of tape needed to make the triangle, which of the following is true?

12 ft

16 ft
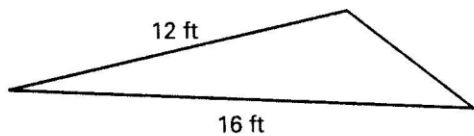
F  $4 < x < 28$

G  $4 < x < 56$
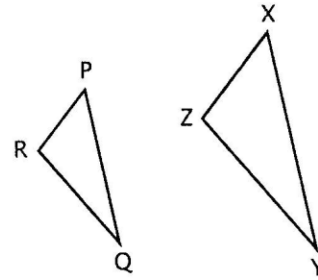
H  $12 < x < 16$

J  $32 < x < 56$

**21** Which similarity statement and postulate or theorem correctly identifies the triangles' relationship?

A  6  12  D
B  9  C  8  E
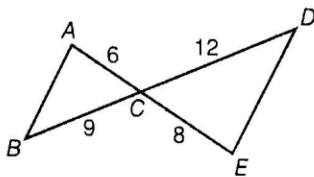
A  $\triangle ABC \sim \triangle CDE$ by SSS Similarity Theorem

B  $\triangle ABC \sim \triangle EDC$ by SAS Similarity Theorem

C  $\triangle ABC \sim \triangle CDE$ by SAS Similarity Theorem

D  $\triangle ABC \sim \triangle EDC$ by AA Similarity Theorem

**22** In the figure below, $\angle P \cong \angle X$.

Which of the following would be sufficient to prove the triangles are similar?

A  $\dfrac{RP}{ZX} = \dfrac{RQ}{XY}$

B  $\dfrac{RP}{ZX} = \dfrac{PQ}{XY}$

C  $\dfrac{RQ}{ZX} = \dfrac{RP}{ZY}$

D  $\dfrac{RP}{ZX} = \dfrac{RQ}{ZY}$

**23** $\triangle JKL$ and $\triangle PQR$ are two triangles such that $\angle K \cong \angle Q$. Which of the following is sufficient to prove the triangles are similar?

F  $\dfrac{JK}{PQ} = \dfrac{KL}{PR}$

G  $JK = PQ$

H  $\dfrac{JK}{PQ} = \dfrac{KL}{QR}$

J  $\angle J$ is right.

**24** In the figure below, $\overline{PQ} \parallel \overline{SR}$.
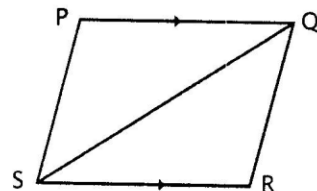
Which additional information would be enough to prove $\triangle PQS \cong \triangle RSQ$?

A  $\overline{PQ} \cong \overline{PS}$

C  $\overline{PQ} \cong \overline{SR}$

B  $\overline{SR} \cong \overline{QR}$

D  $\overline{PS} \cong \overline{QR}$

38

**125** In the figure below, *C* is the midpoint of $\overline{AD}$.



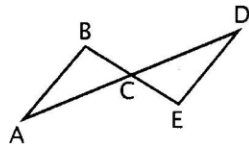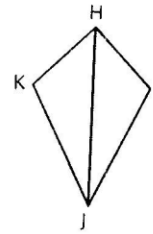Which additional piece of information is needed to prove $\triangle ABC \cong \triangle DEC$?

**A** $\overline{AC} \cong \overline{CD}$

**B** $\overline{AB} \parallel \overline{DE}$

**C** $\angle BCA \cong \angle ECD$

**D** $\overline{AB} \cong \overline{DE}$

**126** In the figure below, quadrilateral *PQRS* is a square and *RT = RU*?
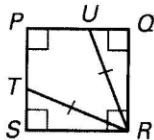


Which theorem or postulate can be used to prove $\triangle RST \cong \triangle RQU$.

**F** SSS

**G** AAS

**H** SAS

**J** HL

**127** In the figure below, $\overline{WZ} \parallel \overline{XY}$.
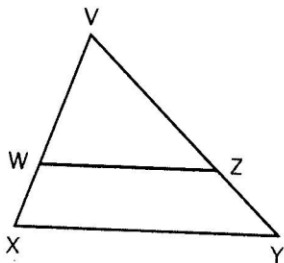


Which theorem or postulate can be used to prove $\triangle VWZ \sim \triangle VXY$?

**A** AAA

**B** AAS

**C** SAS

**D** SSS

**128** In the figure below, $\overline{HJ}$ bisects $\angle KHI$ and $\angle KJI$.



Which theorem or postulate can be used to prove $\triangle HKJ \cong \triangle HIJ$?

**F** ASA

**G** AAS

**H** SAS

**J** SSS

| SOL G.9 |
|---|

The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems.
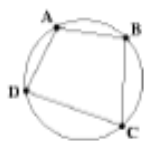
- Solve problems, including real-world problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids.
- Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula.
- Prove the characteristics of quadrilaterals, using deductive reasoning, algebraic, and coordinate methods.
- Prove properties of angles for a quadrilateral inscribed in a circle.

## WHAT I NEED TO KNOW:

| CHARACTERISTICS | WHICH QUADRILATERALS HAVE THIS CHARACTERISTIC | WHAT ARE THE COORDINATE PROOFS FOR THIS CHARACTERISTIC |
|---|---|---|
| **SIDES** | | |
| 1. | | |
| 2. | | |
| 3. | | |
| **ANGLES** | | |
| 1. | | |
| 2. | | |
| **DIAGONALS** | | |
| 1. | | |
| 2. | | |
| 3. | | |
| 4. | | |

| HOW TO PROVE A QUADRILATERAL IS A SPECIFIC FIGURE | | | | | |
|---|---|---|---|---|---|
| PARALLELOGRAM | RECTANGLE | RHOMBUS | SQUARE | ISOSCELES TRAPEZOID | TRAPEZOID |
| | | | | | |



| PROPERTIES OF ANGLES FOR A QUAD INSCRIBED IN A CIRCLE | |
|---|---|

### G.9 PROBLEMS:

**Given: Quadrilateral $ABCD$**
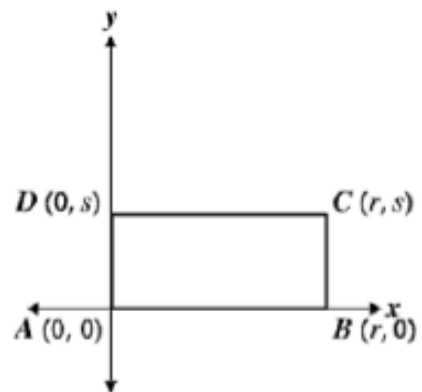


**Which expression proves that $ABCD$ is a rectangle?**

⟩ **A** The length of each diagonal is $\sqrt{r^2 + s^2}$.

⟩ **B** The common midpoint of the diagonals is $\left(\frac{r}{2}, \frac{s}{2}\right)$.

⟩ **C** The slope of $\overline{AC}$ is $\frac{s}{r}$ and the slope of $\overline{BD}$ is $\frac{^-s}{r}$.

⟩ **D** The length of both $\overline{AB}$ and $\overline{CD}$ is $r$ and the length of both $\overline{AD}$ and $\overline{BC}$ is $s$.

---

The diagonals of a rhombus are 10 and 24. Find the perimeter of the rhombus.
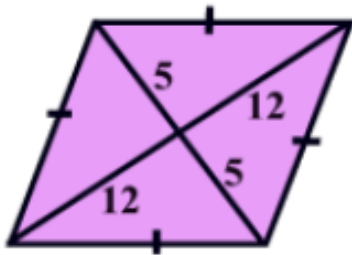


**Find the length of the side of the parallelogram represented by 4x -1.**
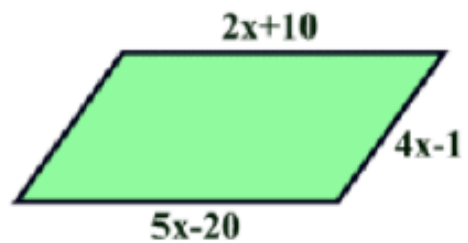
## SOL G.10

The student will solve real-world problems involving angles of polygons.

- Solve real-world problems involving the measures of interior and exterior angles of polygons.
- Identify tessellations in art, construction, and nature.
- Find the sum of the measures of the interior and exterior angles of convex polygon.
- Find the measure of each interior and exterior angle of a regular polygon.
- Find the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon.

## WHAT I NEED TO KNOW:

| POLYGONS | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| DEFINITION OF REGULAR POLYGON | | | | | | | | |
| | | | | | | | | |
| NUMBER OF SIDES GIVEN THE NAME | | | | | | | | |
| TRIANGLE | QUADRILATERAL | PENTAGON | HEXAGON | HEPTAGON | OCTAGON | NONAGON | DECAGON | DODECAGON |
| | | | | | | | | |

| HOW TO FIND THE SUM OF THE INTERIOR AND EXTERIOR OF CONVEX POLYGONS | HOW TO FIND THE MEASURE OF EACH INTERIOR AND EXTERIOR OF A REGULAR POLYGON | HOW TO FIND THE NUMBER OF SIDES OF A REGULAR POLYGON, GIVEN THE MEASURES OF INTERIOR OR EXTERIOR ANGLES OF THE POLYGON |
|---|---|---|
| | | |

| TESSELLATIONS | |
|---|---|
| WHICH REGULAR POLYGONS TESSELLATE | |
| | |
| DEFINITION OF A TESSELLATION | THE RULE ABOUT INTERIOR ANGLES |
| | |

## G.10 PROBLEMS:

This figure is composed of a regular pentagon and a rectangle.
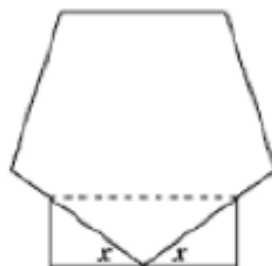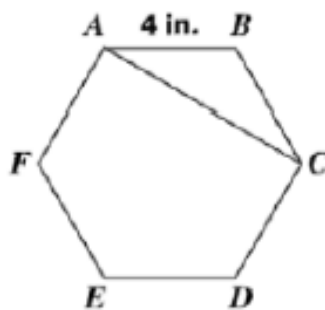


What is the measure of each of the angles identified as $x$ ?

A  36°

B  54°

C  72°

D  108°

# Which of these regular polygons could tessellate a plane?

Square     Pentagon  Octagon   Hexagon   Decagon
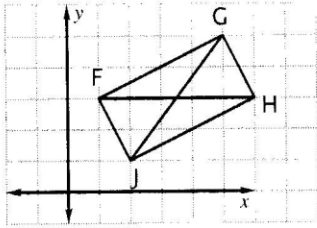
The figure shown is a regular hexagon.



What is the length of the diagonal $AC$ ?

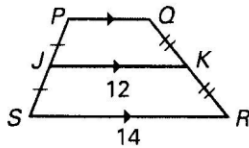A  $4\sqrt{3}$ in.

B  8 in.

C  12 in.

D  $8\sqrt{3}$ in.

**129** The diagram shows parallelogram *FGHJ*. Which statement would prove that parallelogram *FGHJ* is a rectangle?



**F**  $FH = JG$

**G**  $FG = JH$

**H**  $(\text{slope } \overline{FH})(\text{slope } \overline{JG}) = 1$

**J**  $(\text{slope } \overline{FH})(\text{slope } \overline{JG}) = -1$

**130** Quadrilateral *PQRS* is a trapezoid.



What is *PQ*?

**A**  8          **C**  13

**B**  10        **D**  16

**131** Quadrilateral *WXYZ* is a parallelogram. If its diagonals are congruent, which statement must be true?

**F**  Quadrilateral *WXYZ* is a kite.

**G**  Quadrilateral *WXYZ* is a rhombus.

**H**  Quadrilateral *WXYZ* is a rectangle.

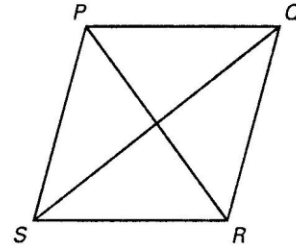**J**  Quadrilateral *WXYZ* is an isosceles trapezoid.

**132** Quadrilateral *ABCD* has vertices at $A(0, 0)$, $B(2, 5)$, $C(5, 2)$ and $D(3, -3)$. What is the most specific name for *ABCD*?

**A**  Trapezoid

**B**  Kite

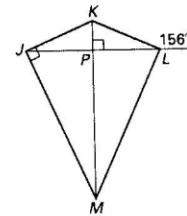**C**  Rhombus

**D**  Parallelogram

**133** Molly cuts quadrilateral *PQRS* into four triangles by cutting along the diagonals. If this produces four congruent triangles, what type of quadrilateral could *PQRS* be?



**F**  Trapezoid

**G**  Kite

**H**  Rhombus

**J**  Rectangle

**134** Quadrilateral *JKLM* is a kite.



If $\angle KJM$ is a right angle, what is the measure of $\angle JML$?

**A**  33°   **B**  48°   **C**  56°   **D**  66°

**135** What is the measure of an exterior angle of a regular pentagon?

**F**  36°

**G**  45°

**H**  72°

**J**  90°

44

**136** Which expression gives the measure for an interior angle of a regular polygon with *n* sides?

A $\dfrac{360}{n}$

B $\dfrac{180}{n}$

C $\dfrac{180(n-2)}{n}$

D $\dfrac{360(n-2)}{n}$

**137** Glenn needs to choose a tile shape that tessellates so that he can tile a floor with a single shape of tile. Which of the following shapes should Glenn *not* select?

F   Scalene triangle

G   Equilateral triangle
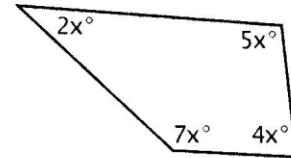
H   Regular hexagon

J   Regular octagon

**138** The coin shown is in the shape of a regular 11-gon. What is the sum of the measures of the interior angles?
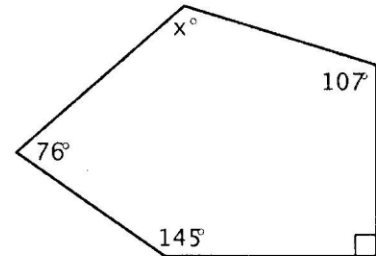


A   1100°

B   1620°

C   1980°

D   3240°

**139** What is the measure of the *smallest* interior angle in the quadrilateral?



F   20°          H   80°

G   40°          J   140°

**140** Veronica is installing carpet in a room shaped like the polygon shown below. She needs to know all of the interior angle measures in the room so that she can cut the carpet. What is *x* in the figure below?
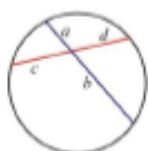


A   32°

B   92°

C   102°

D   122°

## SOL G.11

- Find lengths, angle measures, and arc measures associated with
  – two intersecting chords;
  – two intersecting secants;
  – an intersecting secant and tangent;
  – two intersecting tangents; and
  – central and inscribed angles.
- Calculate the area of a sector and the length of an arc of a circle, using proportions.
- Solve real-world problems associated with circles, using properties of angles, lines, and arcs.
- Verify properties of circles, using deductive reasoning, algebraic, and coordinate methods.
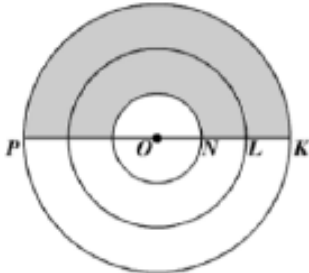
## WHAT I NEED TO KNOW:

### FORMULAS (AREA AND CIRCUMFERENCE ON FORMULA SHEET)
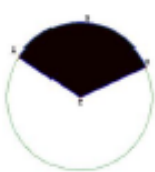
| AREA OF SECTOR | | LENGTH OF AN ARC | |
|---|---|---|---|
|  | |  | |

### LENGTHS

| INTERSECTING INSIDE OF CIRCLE | | INTERSECTING OUTSIDE OF CIRCLE | | | |
|---|---|---|---|---|---|
|  | |  |  | |  | |
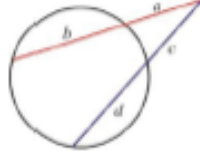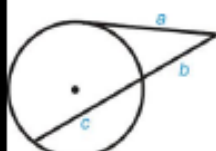
### ANGLE MEASURES
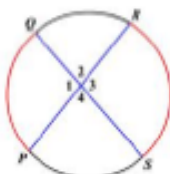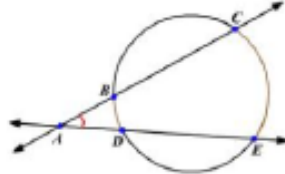
| INSIDE | OUTSIDE | CENTRAL | ON |
|---|---|---|---|
|  |  |  |  |
| | | | |

| CONGRUENT CHORDS | ANGLE BETWEEN TANGENT AND RADIUS |
|---|---|
| | |
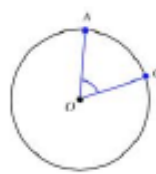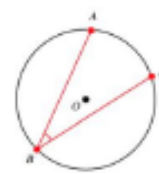
## G.11 PROBLEMS:

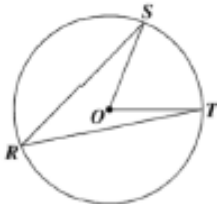Given: Three concentric circles with the center $O$

$$\overline{KL} \cong \overline{LN} \cong \overline{NO}$$
$$KP = 42 \text{ inches}$$

Which is closest to the area of the shaded region?

A  231 sq in.

B  308 sq in.

C  539 sq in.
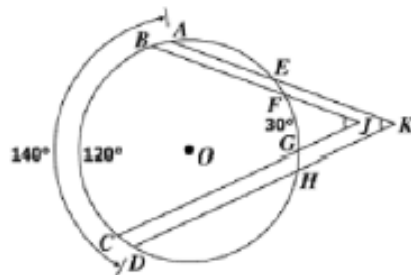
D  616 sq in.

---

In circle $O$, $m\angle SOT = 68°$.

What is $m\angle SRT$ ?

$m\angle SRT = \boxed{\phantom{xx}}°$

Bob divides his circular garden into 10 congruent sectors to plant different types of flowers. The circumference of Bob's garden is 50.5 feet.  What is the area of one sector of Bob's garden?
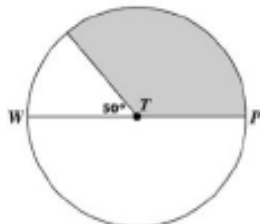
---

In circle $O$, $m\overline{FG} = 30°$, $m\overline{BC} = 120°$, and $\angle J \cong \angle K$.

140°  120°  30°

What is $m\widehat{EH}$ ?

A  35°

B  40°

C  45°

D  50°

---

Given: Circle $T$ with $WP = 36$ centimeters

50°

Which best represents the area of the shaded sector?

A  $117\pi$ cm²

B  $180\pi$ cm²

C  $234\pi$ cm²

D  $468\pi$ cm²

Given:  Circle $M$ with secants $\overrightarrow{AB}$ and $\overrightarrow{AC}$

$m\angle A = 30°$
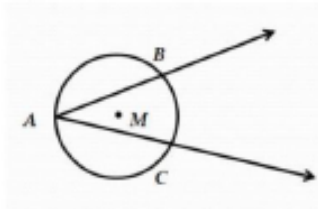
If the length of arc $BC$ is 3 cm, what is the circumference of the circle?

## SOL G.12

The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle.

- Identify the center, radius, and diameter of a circle from a given standard equation.
- Use the distance formula to find the radius of a circle.
- Given the coordinates of the center and radius of the circle, identify a point on the circle.
- Given the equation of a circle in standard form, identify the coordinates of the center and find the radius of the circle.
- Given the coordinates of the endpoints of a diameter, find the equation of the circle.
- Given the coordinates of the center and a point on the circle, find the equation of the circle.
- Recognize that the equation of a circle of given center and radius is derived using the Pythagorean Theorem.

## WHAT I NEED TO KNOW:

| HOW TO USE THE DISTANCE FORMULA TO FIND THE RADIUS OF THE CIRCLE |
|---|
|  |

**ON FORMULA SHEET:**

$(h, k)$  $r$
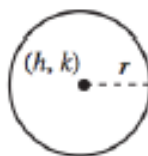
$$(x - h)^2 + (y - k)^2 = r^2$$

**DERIVED FROM PYTHAGOREAN THEOREM!!!**

| HOW TO IDENTIFY THE FOLLOWING GIVEN EQUATION: | | |
|---|---|---|
| CENTER | RADIUS | DIAMETER |
|  |  |  |

| HOW TO FIND THE EQUATION OF THE CIRCLE GIVEN: | |
|---|---|
| COORDINATES OF THE ENDPOINTS OF A DIAMETER | COORDINATES OF THE CENTER AND A POINT ON THE CIRCLE |
|  |  |

| HOW TO IDENTIFY A POINT ON THE CIRCLE GIVEN THE CENTER AND RADIUS | |
|---|---|
| WITHOUT CALCULATOR | WITH CALCULATOR APP |
|  |  |

## G.12 PROBLEMS:

Given: Circle $O$ with diameter $\overline{CD}$
$C(-7, -4)$ and $D(1, 2)$

Create the equation of this circle.

**The Equation of the Circle**

| $\square$ | + | $\square$ | = | $\square$ |

| $(x-3)^2$ | $(x+3)^2$ |
|-----------|-----------|
| $(y-1)^2$ | $(y+1)^2$ |
| 25 | 100 |

Given: Circle $W$
$W(-4, 6)$
Radius $= 10$ units

Which point lies on circle $W$?

A $(0, 4)$

B $(2, 10)$

C $(4, 0)$

D $(6, 16)$

---

The coordinates of the center of a circle are (-2,6). This circle has a diameter of 10 units.

a) What is the equation of the circle?
b) Give the integral coordinates of two points that lie on the circle.

---

The equation of a circle is $(x-3)^2 + (y+4)^2 = 16$.

a) What are the coordinates of the center of the circle?
b) What is the radius of the circle?
c) What is the diameter of the circle?
d) Give the integral coordinates of two points that lie on the circle.

---

Circle O is defined by the equation $x^2 + (y-2)^2 = 25$. Plot the center of circle O and one point with integral coordinates that lies on circle O.
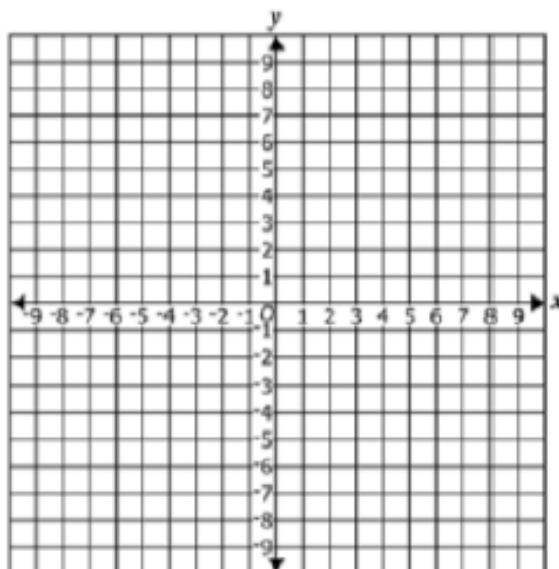
**141** In the figure below, secant $\overline{PQ}$ intersects the circle at $R$ and secant $\overline{SQ}$ intersects the circle at $T$.
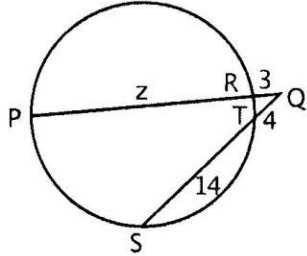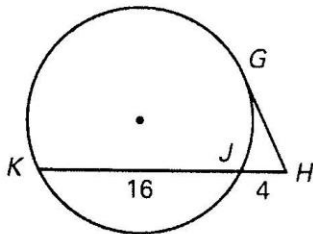


What is the value of $z$?

**F** about 15.67    **H** 21

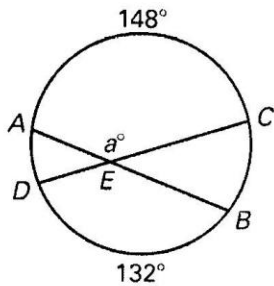**G** about 18.67    **J** 24

**142** In the figure below, secant $\overline{HK}$ intersects the circle at $J$ and $\overline{GH}$ is tangent to the circle at $G$. If $KJ = 16$ and $JH = 4$, what is $GH$?



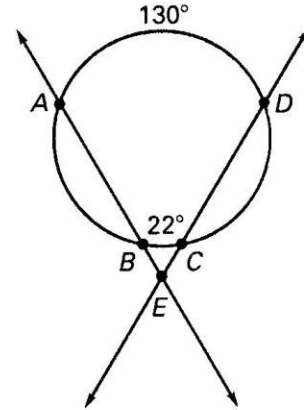**A** $2\sqrt{5}$    **B** 8    **C** $4\sqrt{5}$    **D** 12

**143** What is the value of $a$?

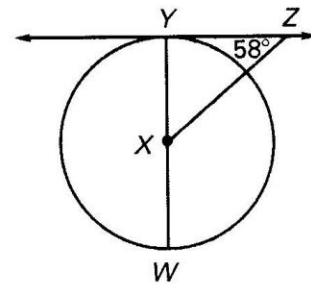

**F** 66    **H** 140

**G** 74    **J** 280

**144** In the figure below, $m\widehat{AD} = 130°$ and $m\widehat{BC} = 22°$. What is the measure of $\angle AED$?



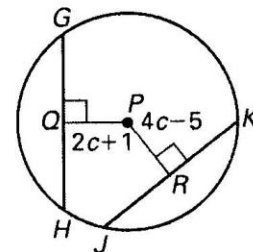**A** 44° **B** 54° **C** 65° **D** 76°

**145** $\overline{WY}$ is a diameter of circle $X$, and $\overline{YZ}$ is tangent to the circle at $Y$. What is $m\angle WXZ$?



**F** 32°    **H** 116°

**G** 58°    **J** 148°

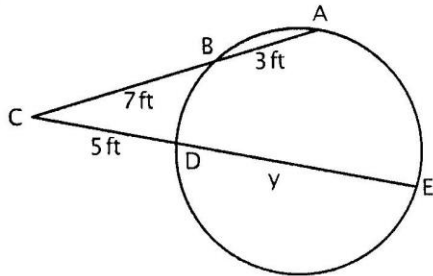**146** In circle $P$ below, $GH = JK = 13$. $PR = 4c - 5$.



What is $PQ$?
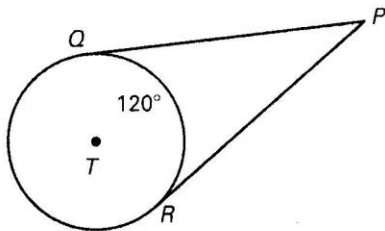
**A** 7 **B** 5 **C** 3 **D** 2

**147** Two children are sitting at points $A$ and $E$ at the edge of the circular sandbox shown below. Their mother is standing at point $C$. What is the value of $y$?


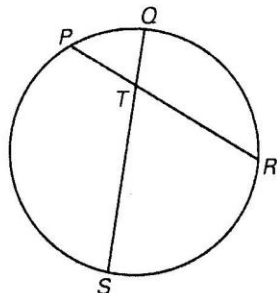
A   4.2 ft

B   5 ft

C   9 ft

D   9.8 ft

**148** A satellite orbiting Earth takes a picture that shows Earth from point $Q$ to point $R$ in the diagram below. What is the angle of the camera lens, represented by $\angle QPR$?



F   30°

G   45°

H   60°

J   75°

**149** A circular cake is cut into four sections as shown in the diagram below. If $m\widehat{SP} = 135°$ and $m\angle STR = 70°$, what is $m\widehat{QR}$?



F   55°

G   85°

H   110°

J   122.5°

**150** A drop leaf table is made by cutting and hinging two parallel, congruent chords in a circular tabletop. If the length of the chords is 37 inches and the distance between the two cords is 26 inches, what is the radius of the original tabletop?
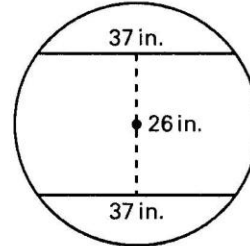


A   22.6 in.

B   26 in.

C   28.3 in.

D   31.5 in.

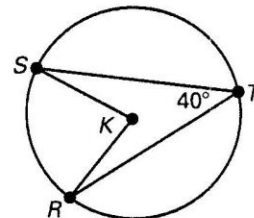**151** Caitlyn is making a circular stained glass window out of four pieces of glass as shown below. What is the value of $x$?
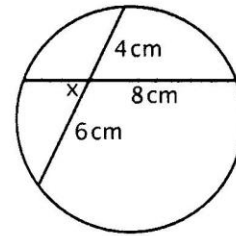


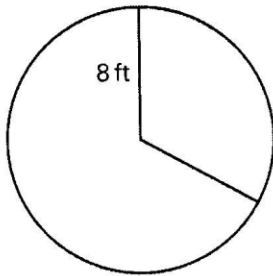F   3 cm

G   4 cm

H   5 cm

J   6 cm

**152** The design below is part of a company logo. What is $m\angle SKR$?



A   40°

B   60°

C   80°

D   90°

**153** Maria is planting a garden in the shaded area of the circle shown below. The area of her garden is 68.1 ft². She wants to place garden edging along the curved edge of her garden. How much edging will she need? Round your answer to the nearest tenth.



8 ft

**A**  15.2 ft          **C**  17.0 ft
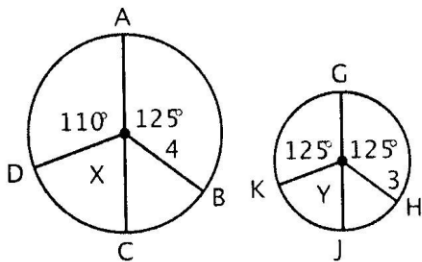
**B**  16.5 ft          **D**  18.3 ft

**154** A circular pizza has a diameter of 14 inches and is cut into 8 equal slices. To the nearest tenth of a square inch, which answer represents the area of one slice?

**F**  615.8 in.²        **H**  22.0 in.²
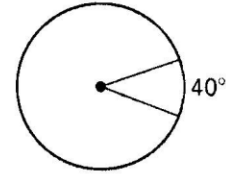
**G**  44.0 in.²         **J**  19.2 in.²

**155** In the figure below, $\overline{AC}$ is a diameter of circle $X$, and $\overline{GJ}$ is a diameter of circle $Y$. Which statement is true?



A
110° | 125°
4
D     X     B
K     Y     H
125° | 125°
3
C
G
J

**A**  $\overset{\frown}{GH} \cong \overset{\frown}{BCD}$          **C**  $\overset{\frown}{AB} \cong \overset{\frown}{BCD}$

**B**  $\overset{\frown}{AB} \cong \overset{\frown}{GK}$          **D**  $\overset{\frown}{GK} \cong \overset{\frown}{KJH}$
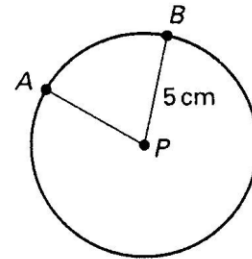
**156** The radius of the circle shown below is 11 centimeters. What is the area of the shaded sector?



40°

**F**  9.5 cm²

**G**  28.3 cm²

**H**  42.2 cm²

**J**  69.1 cm²

**157** The area of sector $APB$ is 13.5 cm². What is the length of $\overset{\frown}{AB}$? Round to the nearest tenth.



B
A
5 cm
P

**A**  5.4 cm

**B**  7.9 cm

**C**  8.6 cm

**D**  10.8 cm

52

**158** An equation of a circle is $(x - 1)^2 + (y + 4)^2 = 8$. What are the center and radius of the circle?

A  Center $(1, -4)$; radius 4

B  Center $(-1, 4)$; radius 4

C  Center $(1, -4)$; radius $2\sqrt{2}$

D  Center $(-1, 4)$; radius $2\sqrt{2}$

**159** A circle is centered at $(-2, 3)$ and contains point $(5, 9)$. Which is an equation of the circle?

A  $(x + 2)^2 + (y - 3)^2 = 13$

B  $(x - 5)^2 + (y - 9)^2 = 85$

C  $(x + 2)^2 + (y - 3)^2 = 85$

D  $(x - 2)^2 + (y - 3)^2 = 45$

**160** An equation of a circle is $(x - 4)^2 + (y + 5)^2 = 36$. What are the center and diameter of the circle?

F  Center $(4, -5)$; diameter 6

G  Center $(4, -5)$; diameter 12

H  Center $(-4, 5)$; diameter 6

J  Center $(-4, 5)$; diameter 12

**161** Which is an equation of a circle with diameter 8 and center at $(1, 2)$?

A  $(x + 1)^2 + (y + 2)^2 = 64$

B  $(x + 1)^2 + (y + 2)^2 = 16$

C  $(x - 1)^2 + (y - 2)^2 = 64$

D  $(x - 1)^2 + (y - 2)^2 = 16$

**162** Which is an equation of a circle with radius 3 and center at $(4, -1)$?

F  $(x - 4)^2 + (y + 1)^2 = 9$

G  $(x + 4)^2 + (y - 1)^2 = 9$

H  $(x - 1)^2 + (y + 4)^2 = 9$

J  $(x + 1)^2 + (y - 4)^2 = 9$

**163** The endpoints of a diameter of a circle are $(11, 13)$ and $(8, 17)$. Which is an equation of the circle?

F  $(x - 8)^2 + (y - 17)^2 = 5$

G  $(x - 11)^2 + (y - 13)^2 = 25$

H  $(x - 9.5)^2 + (y - 15)^2 = 25$

J  $(x - 9.5)^2 + (y - 15)^2 = 6.25$

**164** An equation of a circle is $(x - 3)^2 + (y + 15)^2 = 50$. What is the radius of the circle?

A  $5\sqrt{2}$

B  $10\sqrt{2}$

C  25

D  50

**165** A circle is centered at $(1, -6)$ and has a radius of 10. Which of the following is a point on the circle?

F  $(-11, 0)$

G  $(-5, 2)$

H  $(3, 5)$

J  $(11, -6)$

## SOL G.13

The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.

- Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.
- Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.
- Solve problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.
- Calculators may be used to find decimal approximations for results.

## WHAT I NEED TO KNOW:
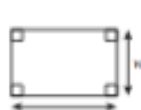## HOW TO USE THE FORMULAS ON THE FORMULA SHEET

$A = \frac{1}{2}bh$

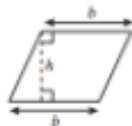$A = \frac{1}{2}ab \sin C$

$p = 4s$
$A = s^2$

$p = 2l + 2w$
$A = lw$

$C = 2\pi r$
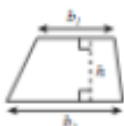$C = \pi d$
$A = \pi r^2$

$A = bh$
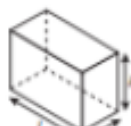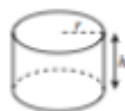
$A = \frac{1}{2}h(b_1 + b_2)$

$V = Bh$
$L.A. = hp$
$S.A. = hp + 2B$

$V = lwh$
$S.A. = 2lw + 2lh + 2wh$

$V = \pi r^2 h$
$L.A. = 2\pi rh$
$S.A. = 2\pi r^2 + 2\pi rh$

$V = \frac{4}{3}\pi r^3$
$S.A. = 4\pi r^2$

$V = \frac{1}{3}\pi r^2 h$
$L.A. = \pi rl$
$S.A. = \pi r^2 + \pi rl$

$V = \frac{1}{3}Bh$
$L.A. = \frac{1}{2}lp$
$S.A. = \frac{1}{2}lp + B$

| KEY WORDS FOR VOLUME | THE DIFFERENCE BETWEEN B AND b | KEY WORDS FOR SURFACE AREA |
|---|---|---|
| | | |

## WATCH YOUR UNITS!!!

## SOMETIMES YOU MAY NEED TO ADD VOLUMES AND SOMETIMES YOU MAY NEED TO SUBTRACT!

A cylinder has a volume of $300\pi$ cubic centimeters and a base with a circumference of $10\pi$ centimeters. What is the height of the cylinder?

A  30 cm

B  15 cm

C  12 cm

D  3 cm

---

Two cylinders, a sphere, and a cone are shown. Select the two objects with the same volume.



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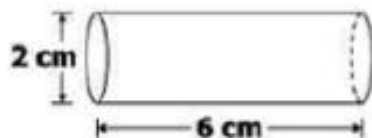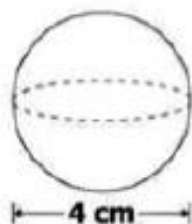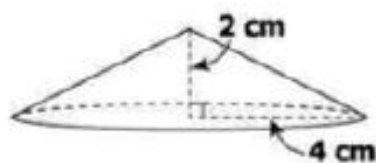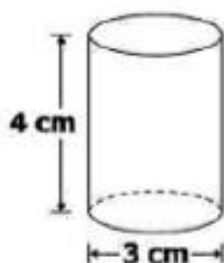
A cone has a slant height of 10 centimeters and a lateral area of $60\pi$ square centimeters. What is the volume of a sphere with a radius equal to that of the cone?
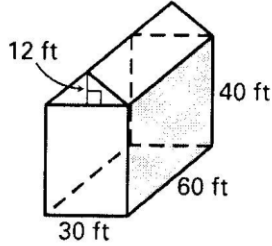
---

A fish tank in the shape of a rectangular prism has these dimensions:

- length $= 20$ inches
- width $= 10$ inches
- height $= 12$ inches

What is the volume of water in the tank when it is $\dfrac{4}{5}$ full?

**166** José wants to calculate the volume of air in a building, shown below, so that he can decide on the size of a new furnace. What is the volume of the building?
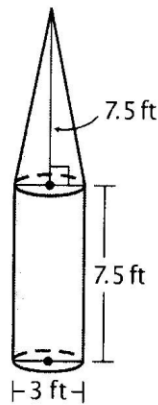
12 ft
40 ft
60 ft
30 ft

F   8280 ft³        H   72,000 ft³

G   82,800 ft³      J   93,600 ft³

**167** What is the surface area of a sphere with a radius of 20 centimeters?

A   5024 cm²        C   20,096 cm²

B   6400 cm²        D   33,493 cm²

The diagram represents a sculpture in an art museum. What is the surface area of the sculpture? Round your answer to the nearest tenth. Use 3.14 for $\pi$.
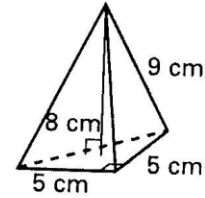
7.5 ft
7.5 ft
3 ft

**168**

F   113.7 ft²       H   241.7 ft²

G   127.9 ft²       J   301.8 ft²

**169** Find the volume of the pyramid. Round to the nearest tenth.

9 cm
8 cm
5 cm
5 cm
5 cm

A   141.4 cm³

B   33.3 cm³

C   47.1 cm³

D   37.5 cm³

**170** What is the volume of this solid?

1 yd
2 yd
3 yd
3 yd
5 yd
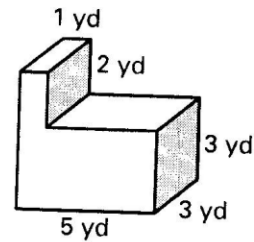
F   14 yd³          H   169 yd³

G   51 yd³          J   270 yd³

**171** What is the volume of the sphere in cubic meters? Use 3.14 for $\pi$. Round your answer to the nearest whole number.

4 m
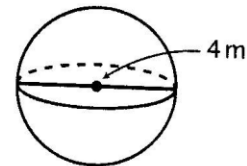
A   33 m³

B   50 m³

C   201 m³

D   268 m³

## SOL G.14

The student will use similar geometric objects in two- or three-dimensions to
  a) compare ratios between side lengths, perimeters, areas, and volumes;
  b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
  c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
  d) solve real-world problems about similar geometric objects.

• Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures.
• Describe how changes in one or more dimensions affect other derived measures (perimeter, area, total surface area, and volume) of an object.
• Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of an object.
• Solve real-world problems involving measured attributes of similar objects.

## WHAT I NEED TO KNOW:

| RATIOS | | | |
|--------|--------|--------|--------|
| LENGTHS | PERIMETERS | SURFACE AREA | VOLUME |
| | | | |

A rectangular prism has a volume of 36 cm$^3$.

a) If the height of the prism is tripled and the other dimensions do not change, what is the volume of the new rectangular prism?

b) If all dimensions of the original rectangular prism are tripled, what is the volume of the new rectangular prism?

The ratio of the volume of two spheres is 8:27. What is the ratio of the lengths of the radii of these two spheres?

☐ : ☐

| 1 | 2 | 3 | 4 | 6 | 8 | 9 | 13 | 19 | 27 |

A cylinder has a surface area of 96 square inches. If all dimensions of this cylinder are multiplied by $\frac{1}{2}$ to create a new cylinder, what will be the surface area of the new cylinder?

**172** ABC Moving Company makes shipping crates in two sizes that are similar rectangular prisms. If the ratio of volumes of the crates is 5 : 21, approximately what is the ratio of side lengths of the crates?

F  1.7 : 2.8

G  1.7 : 7

H  2.2 : 4.6

J  2.5 : 10.5

**173** The surface area of Sphere A is 27 m². The surface area of Sphere B is 48 m². What is the ratio of the diameter of Sphere A to the diameter of Sphere B, expressed as a decimal? Round your answer to the nearest hundredth.

A  0.83

B  0.56

C  0.55

D  0.75

**174** Sam is making a scale model of an Egyptian pyramid. The ratio of the surface area of his model to the surface area of the pyramid is 1 : 100. What is the ratio of the volume of the model to the volume of the pyramid?

F  1 : 10

G  1 : 200

H  1 : 500

J  1 : 1000

**175** The triangles shown below are similar. What is the ratio of the perimeter of the larger right triangle to the perimeter of the smaller right triangle?
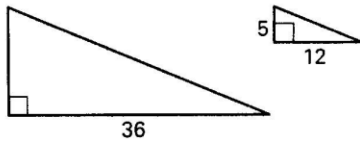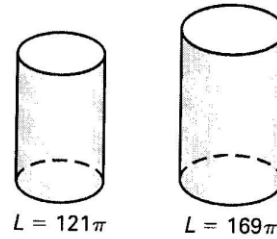


A  36 : 5

B  9 : 1

C  3 : 1

D  36 : 3

**176** The perimeters of two squares are in a ratio of 5 to 11. What is the ratio between the areas of the two squares?

F  15 to 33

G  2 to 3

H  5 to 11

J  25 to 121

**177** The lateral areas of two similar cylindrical juice containers are shown. What is the ratio of their volumes?



L = 121π    L = 169π

A  11 : 13    C  121 : 169

B  33 : 39    D  1331 : 2197

**178** An architect is designing a building in the shape of a triangular prism. If the ratio of the height of her scale model to the height of the actual building is 1 : 98, what is the ratio of the volume of the scale model to the volume of the building?

F  1 : 196    H  1 : 9604

G  1 : 294    J  1 : 941,192

**179** Alana has one large toy box and one small toy box. Both of the toy boxes are in the shape of a cube. What is the scale factor of the surface areas of the toy boxes?



14 in.    28 in.

A  1 : 2    C  1 : 4

B  2 : 3    D  1 : 8
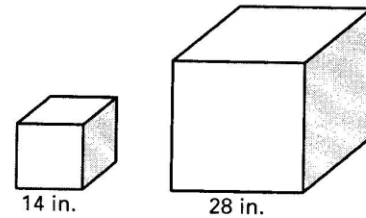
58

**180** The dimensions of a sphere are increased by a scale factor of 4. The surface area of the original sphere is about 314 cm². What is the surface area of the larger sphere?

A 1256 cm²

B 2512 cm²

C 3768 cm²

D 5024 cm²

**181** Mr. Gonzalez needs to increase the space he rents at a boat yard. He currently rents a rectangular storage space of 6000 cubic feet. If he increases the dimensions of the storage space 1.5 times, what will be the volume of the new storage space?

F 9000 ft³

G 13,500 ft³

H 20,250 ft³

J 27,000 ft³

**182** A large gazebo is shaped like a regular octagon. Its sides are 12 feet and it has an area of about 696 square feet. Find the area of a similar gazebo that has a side of length 8 feet. Round to the nearest tenth.

A 130.5 ft²

B 309.3 ft²

C 116 ft²

D 1566 ft²

**183** Two triangles are similar with a scale factor of 4. How many times greater is the area of the larger triangle compared to the area of the smaller triangle?

A 2

B 4

C 8

D 16

**184** A scale model of the ramp shown below has a volume of 0.34375 ft³. What is the height of the scale model ramp?
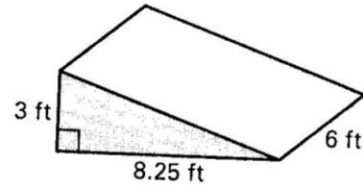


A 0.5 ft

B $\frac{\sqrt{6}}{12}$ ft

C 1 ft

D 3 ft

**185** A rectangle has a width of 8 centimeters and an area of 160 centimeters. A similar rectangle has an area of 250 centimeters. What are the dimensions of the larger rectangle?

F 8 cm by 31.25 cm

G 10 cm by 25 cm

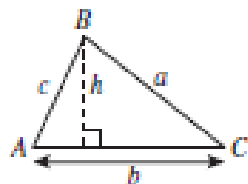H 12.5 cm by 20 cm

J 15 cm by $16\frac{2}{3}$ cm

**186** Two similar cones are shown below. The ratio of their surface areas is 1:25. If the radius of the smaller cone is 4 cm, what is the radius of the larger cone?
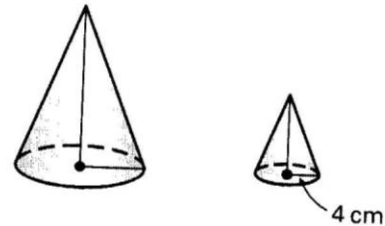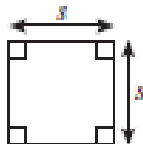


A 10 cm

B 20 cm

C 25 cm

D 100 cm

# Geometry Formula Sheet
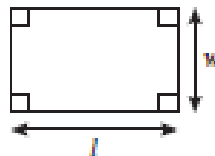## 2009 *Mathematics Standards of Learning*

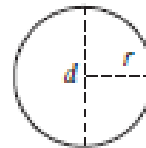### Geometric Formulas

$A = \dfrac{1}{2}bh$

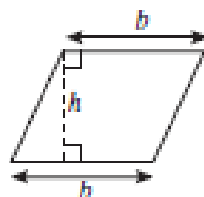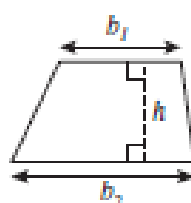$A = \dfrac{1}{2}ab \sin C$

$p = 4s$

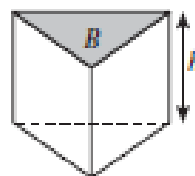$A = s^2$

$p = 2l + 2w$
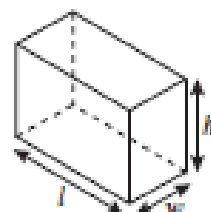
$A = lw$

$C = 2\pi r$

$C = \pi d$

$A = \pi r^2$

$A = bh$

$A = \dfrac{1}{2}h(b_1 + b_2)$

$V = Bh$
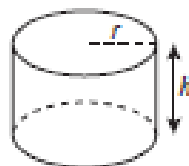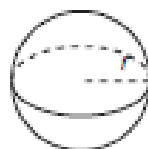
$L.A. = hp$

$S.A. = hp + 2B$

$V = lwh$

$S.A. = 2lw + 2lh + 2wh$

$V = \pi r^2 h$

$L.A. = 2\pi rh$
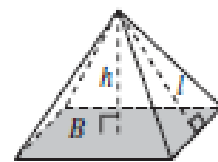
$S.A. = 2\pi r^2 + 2\pi rh$

$V = \dfrac{4}{3}\pi r^3$

$S.A. = 4\pi r^2$

$V = \dfrac{1}{3}\pi r^2 h$

$L.A. = \pi rl$

$S.A. = \pi r^2 + \pi rl$

$V = \dfrac{1}{3}Bh$
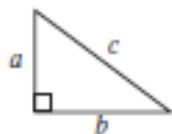
$L.A. = \dfrac{1}{2}lp$

$S.A. = \dfrac{1}{2}lp + B$

### Abbreviations

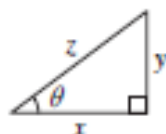| | |
|---|---|
| Area | A |
| Area of Base | B |
| Circumference | C |
| Lateral Area | L.A. |
| Perimeter | P |
| Surface Area | S.A. |
| Volume | V |

# Geometry Formula Sheet
## 2009 *Mathematics Standards of Learning*
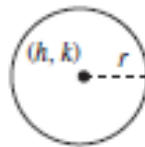
### Geometric Formulas

$$a^2 + b^2 = c^2$$

$$\text{Sin } \theta = \frac{y}{z}$$

$$\text{Cos } \theta = \frac{x}{z}$$

$$\text{Tan } \theta = \frac{y}{x}$$

$$(x - h)^2 + (y - k)^2 = r^2$$

**Pi**

$$\pi \approx 3.14$$

$$\pi \approx \frac{22}{7}$$

**Quadratic Formula:**

$$x = \frac{^-b \pm \sqrt{b^2 - 4ac}}{2a}, \text{ where } ax^2 + bx + c = 0 \text{ and } a \neq 0$$

### Geometric Symbols

| Example | Meaning |
|---------|---------|
| $m\angle A$ | measure of angle $A$ |
| $AB$ | length of line segment $AB$ |
| $\overrightarrow{AB}$ | ray $AB$ |
| | right angle |
| $\overline{AB} \parallel \overline{CD}$ | Line $AB$ is parallel to line $CD$. |
| $\overline{AB} \perp \overline{CD}$ | Line segment $AB$ is perpendicular to line segment $CD$. |
| $\angle A \cong \angle B$ | Angle $A$ is congruent to angle $B$. |
| $\triangle ABC \sim \triangle DEF$ | Triangle $ABC$ is similar to triangle $DEF$. |
| | Similarly marked segments are congruent. |
| | Similarly marked angles are congruent. |