# SOL G.1

The student will construct and judge the validity of a logical argument consisting of a set of premises and a conclusion. This will include
a) identifying the converse, inverse, and contrapositive of a conditional statement;
b) translating a short verbal argument into symbolic form;
c) using Venn diagrams to represent set relationships; and
d) using deductive reasoning.

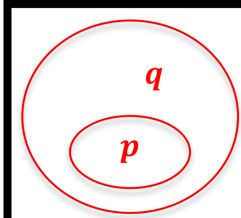
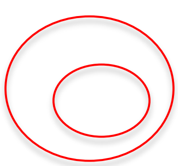
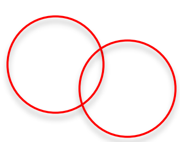
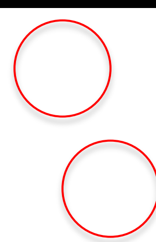
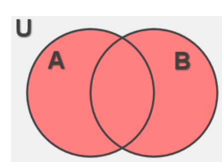
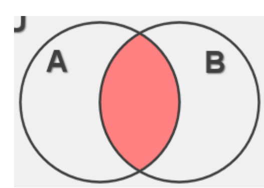
- Identify the converse, inverse, and contrapositive of a conditional statement.
- Translate verbal arguments into symbolic form, such as ($p \rightarrow q$) and ($\sim p \rightarrow \sim q$).
- Determine the validity of a logical argument.
- Use valid forms of deductive reasoning, including the law of syllogism, the law of the contrapositive, the law of detachment, and counterexamples.
- Select and use various types of reasoning and methods of proof, as appropriate.
- Use Venn diagrams to represent set relationships, such as intersection and union.
- Interpret Venn diagrams.
- Recognize and use the symbols of formal logic, which include $\rightarrow$, $\leftrightarrow$, $\sim$, $\therefore$, $\wedge$, and $\vee$.

## WHAT I NEED TO KNOW:

| CONDITIONAL STATEMENTS | | |
|---|---|---|
| converse | inverse | contrapositive |
| Switch hypothesis & conclusion | Negate hypothesis & conclusion | Switch & Negate hypothesis & conclusion |
| WHICH IS LOGICALLY EQUIVALENT TO CONDITIONAL? | | |
| Contrapositive is logically equivalent to conditional. | | |

| LAWS | | |
|---|---|---|
| 1. Detachment | 2. Syllogism | 3. Contrapositive |
| $p \rightarrow q$<br>$\underline{p}$<br>$\therefore q$ | $p \rightarrow q$<br>$\underline{q \rightarrow r}$<br>$\therefore p \rightarrow r$ | $p \rightarrow q$<br>$\underline{\sim q}$<br>$\therefore \sim p$ |

| Symbols | |
|---|---|
| $\rightarrow$ | conditional (if, then) |
| $\leftrightarrow$ | biconditional (if, and only if) |
| $\sim$ | not |
| $\therefore$ | therefore |
| $\wedge$ | and |
| $\vee$ | or |

| Venn diagrams | | | | | |
|---|---|---|---|---|---|
| If p, then q. | ALL | SOMETIMES | NO | UNION OF 2 SETS | INTERSECTION OF 2 SETS |
|  |  |  |  |  |  |

| COUNTEREXAMPLE: | Give an example where hypothesis is true, and conclusion is false. |
|---|---|

## G.1 PROBLEMS:

Let $m$ represent:

*Angle $A$ is obtuse.*

Let $n$ represent:

*Angle $B$ is obtuse.*

Which is a symbolic representation of the following argument?

*Angle $A$ is obtuse if and only if Angle $B$ is obtuse.*

*Angle $A$ is obtuse or Angle $B$ is obtuse.*

*Therefore, Angle $A$ is obtuse and Angle $B$ is obtuse.*

**A.** $m \rightarrow n$
$m \wedge n$
$\therefore m \vee n$

**B.** $m \rightarrow n$
$m \vee n$
$\therefore m \wedge n$

**C.** $m \leftrightarrow n$
$m \wedge n$
$\therefore m \vee n$

**D.** $m \leftrightarrow n$
$m \vee n$
$\therefore m \wedge n$

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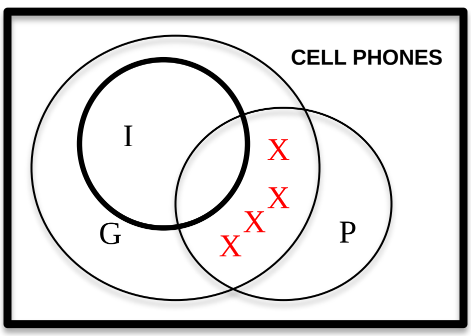
The Venn diagram represents the set of cell phones in a store.

- Let P represent the cell phones that take pictures.
- Let I represent the cell phones that connect to the Internet.
- Let G represent the cell phones that have games.

Identify each region of the Venn diagram that represents the cell phones that only take pictures and have games.

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Let $p$ represent

$\angle A$ is acute.

Let $q$ represent

$\angle B$ is acute.

Create a symbolic representation of the following argument.

| | |
|---|---|
| $\angle A$ is acute if and only if $\angle B$ is acute. | $p \leftrightarrow q$ |
| $\angle A$ is acute or $\angle B$ is acute. | $p \vee q$ |
| Therefore, $\angle A$ is acute and $\angle B$ is acute. | $\therefore p \wedge q$ |

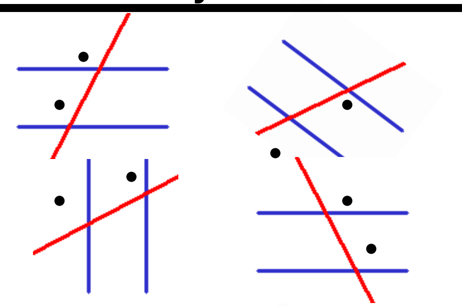
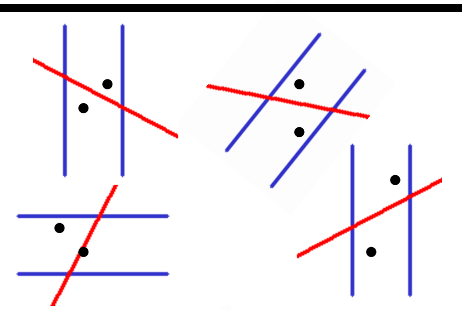
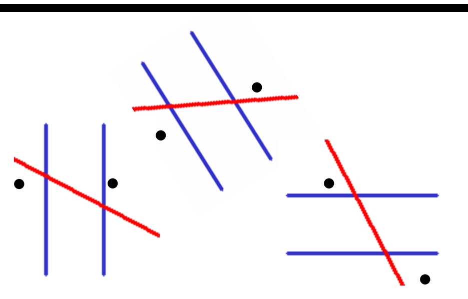
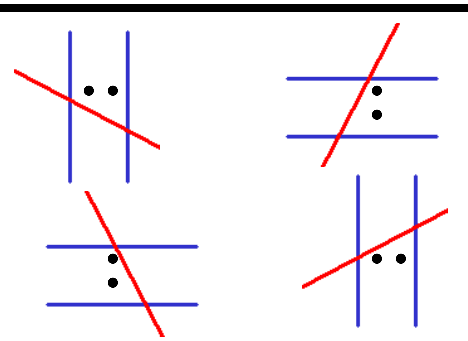
| | | | | | |
|---|---|---|---|---|---|
| $p \rightarrow q$ | $p \leftrightarrow q$ | $p \wedge q$ | $p \vee q$ | $\therefore p \wedge q$ | $\therefore p \vee q$ |

## SOL G.2

The student will use the relationships between angles formed by two lines cut by a transversal to
a) determine whether two lines are parallel;
b) verify the parallelism, using algebraic and coordinate methods as well as deductive proofs;
c) solve real-world problems involving angles formed when parallel lines are cut by a transversal.

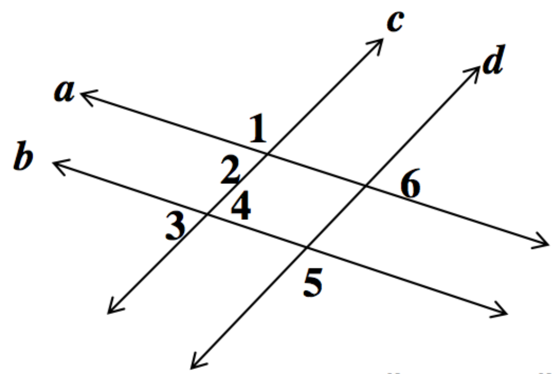
- Use algebraic and coordinate methods as well as deductive proofs to verify whether two lines are parallel.
- Solve problems by using the relationships between pairs of angles formed by the intersection of two parallel lines and a transversal including corresponding angles, alternate interior angles, alternate exterior angles, and same-side (consecutive) interior angles.
- Solve real-world problems involving intersecting and parallel lines in a plane.

## WHAT I NEED TO KNOW:

| Special ANGLE PAIR NAMES | WHERE THEY ARE LOCATED on parallel lines cut by a transversal | ARE THEY CONGRUENT OR SUPPLEMENTARY? | How to set up an equation to solve for x |
|---|---|---|---|
| Corresponding angles |  | congruent | ______ = ____ |
| Alternate interior angles |  | congruent | ______ = ____ |
| Alternate exterior angles |  | congruent | ______ = ____ |
| Same-side (consecutive) interior angles |  | supplementary | ______+_____= 180 |

## G.2 PROBLEMS:

Lines $a$ and $b$ intersect lines $c$ and $d$.




Which statement could be used to prove $a \parallel b$ and $c \parallel d$?

A. $\angle 1$ and $\angle 2$ are supplementary and $\angle 5 \cong \angle 6$

B. $\angle 1 \cong \angle 3$ and $\angle 3 \cong \angle 5$

C. $\angle 3$ and $\angle 5$ are supplementary, and $\angle 5$ and $\angle 6$ are supplementary

D. $\angle 3 \cong \angle 4$ and $\angle 2 \cong \angle 6$

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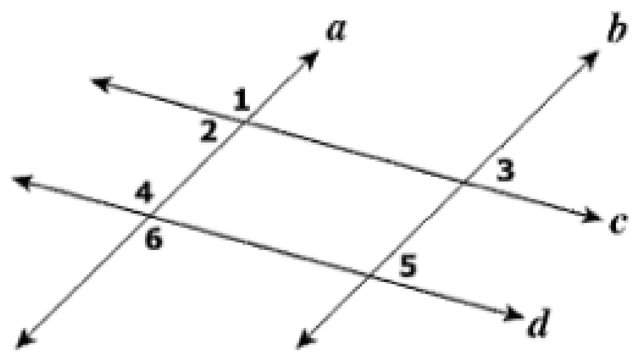
Lines $a$ and $b$ intersect lines $c$ and $d$.




Which of the following statements could be used to prove that $a \parallel b$ and $c \parallel d$ ?

A $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 5$

B $\angle 1 \cong \angle 6$, $\angle 4$ and $\angle 5$ are supplementary

C $\angle 1 \cong \angle 4$, $\angle 1$ and $\angle 2$ are supplementary

D $\angle 1$ and $\angle 3$ are supplementary, $\angle 1$ and $\angle 6$ are supplementary

# SOL G.3

The student will use pictorial representations, including computer software, constructions, and coordinate methods, to solve problems involving symmetry and transformation.

a) investigating and using formulas for finding distance, midpoint, and slope;
b) applying slope to verify and determine whether lines are parallel or perpendicular;
c) investigating symmetry and determining whether a figure is symmetric with respect to a line or a point; and
d) determining whether a figure has been translated, reflected, rotated, or dilated, using coordinate methods.

- Find the coordinates of the midpoint of a segment, using the midpoint formula.
- Use a formula to find the slope of a line.
- Compare the slopes to determine whether two lines are parallel, perpendicular, or neither.
- Determine whether a figure has point symmetry, line symmetry, both, or neither.
- Given an image and preimage, identify the transformation that has taken place as a reflection, rotation, dilation, or translation.
- Apply the distance formula to find the length of a line segment when given the coordinates of the endpoints.

## WHAT I NEED TO KNOW:

### FORMULAS

| DISTANCE | MIDPOINT | SLOPE | ENDPOINT |
|---|---|---|---|
| $\sqrt{a^2+b^2}$<br>or<br>$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ | SAD<br>(Stack Em, Add Em, Divide Em by 2)<br>OR<br>$\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)$ | $\frac{RISE}{RUN}$<br>$\frac{y_2-y_1}{x_2-x_1}$<br>$\frac{\Delta y}{\Delta x}$ | DMSE<br>(Double the Midpoint, Subtract the Endpoint) |

### HOW TO TELL WHETHER LINES ARE PARALLEL OR PERPENDICULAR

| PARALLEL LINES | PERPENDICULAR LINES |
|---|---|
| Slopes are equal | Slopes are opposite reciprocals.<br>Slopes have product of -1. |

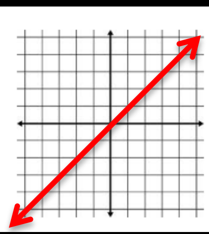
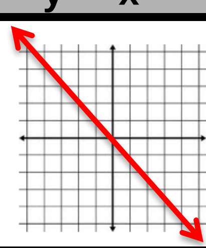
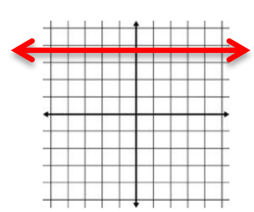
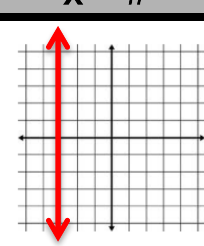
### SYMMETRY

| LINE | POINT |
|---|---|
| fold over a line and get a match | figure looks the same when upside down |

### TRANSFORMATIONS

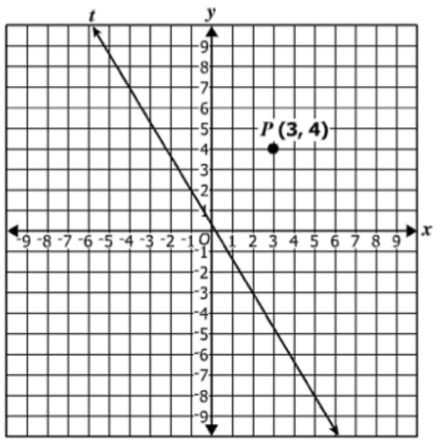
| TRANSLATION | REFLECTION | ROTATION | DILATION |
|---|---|---|---|
| slide<br>$(x,y)\to(x\pm a, y\pm b)$ | flip<br>over x-axis<br>$(x,y)\to(x,-y)$<br>over y-axis<br>$(x,y)\to(y,x)$<br>over y = x<br>$(x,y)\to(y,x)$<br>over y = -x<br>$(x,y)\to(-y,-x)$<br>over origin<br>$(x,y)\to(-x,-y)$ | turn<br>90° rotation:<br>$(x,y)\to(-y,x)$<br>180° rotation:<br>$(x,y)\to(-x,-y)$<br>270° rotation:<br>$(x,y)\to(y,-x)$ | bigger/smaller<br>If center of rotation is origin:<br>$(x,y)\to(kx,ky)$ |

### WHAT THE FOLLOWING GRAPHS LOOK LIKE

| y = x | y = -x | y = # | x = # |
|---|---|---|---|
|  |  |  |  |

## G.3 PROBLEMS:

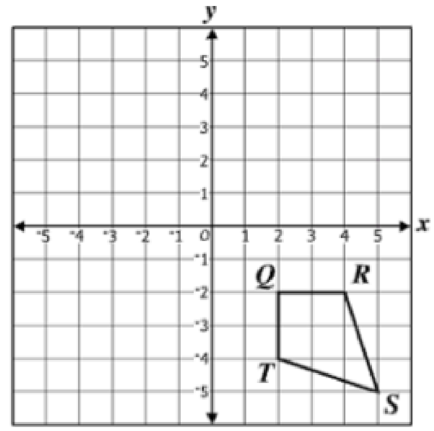
**Line t contains the points (-4,7) and (5,-8). Plot a point other than point P with integral coordinates that lies on a line that is parallel to t and passes through point P.**




(6,-1) or (0,9)

Quadrilateral $QRST$ is to be reflected over the line $y = {}^{-}x$.

What are the coordinates of point $T'$ after this reflection?




**A** $({}^{-}4, 2)$

**B** $({}^{-}2, {}^{-}4)$

**C** $(2, 4)$

**D** $(4, {}^{-}2)$

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**Given: Triangle $ABC$ with vertices located at**

**$A$(1, 1), $B$(2, -3), and $C$(-1, -4).**

**(-4,-1)**

**Triangle $ABC$ will be reflected over the line $y = x$. What will be the integral coordinates of point $C'$ after this transformation?**

$C'$( ☐ , ☐ )

---

**Line $a$ passes through points with coordinates (-4, 5) and (2, -2).**

**What is the slope of a line perpendicular to line $a$?**

**Slope of perpendicular line =** ☐ $\frac{6}{7}$

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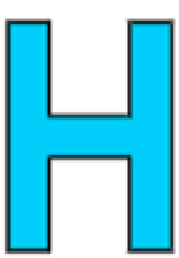
**What is the total number of lines of symmetry for this figure?**



**2**

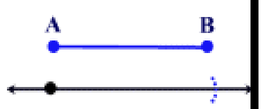
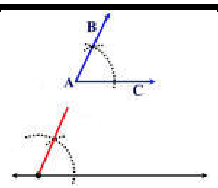
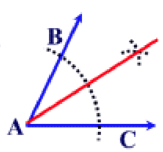
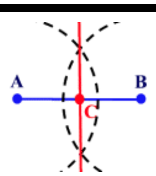
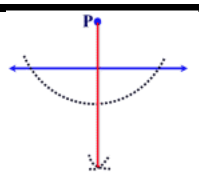
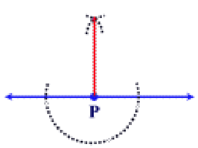
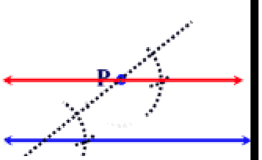
## SOL G.4

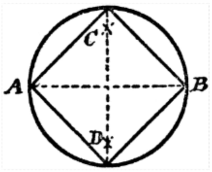
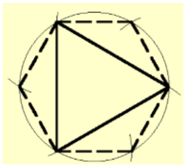
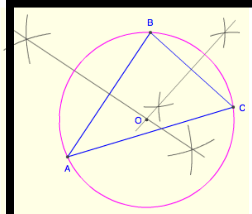
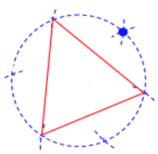
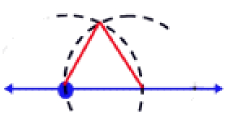
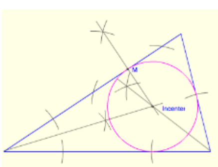
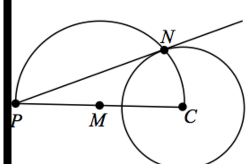
**The student will construct and justify the constructions of**
- a) a line segment congruent to a given line segment;
- b) the perpendicular bisector of a line segment;
- c) a perpendicular to a given line from a point not on the line;
- d) a perpendicular to a given line at a given point on the line;
- e) the bisector of a given angle,
- f) an angle congruent to a given angle; and
- g) a line parallel to a given line through a point not on the given line.

- **Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.**
- **Construct the inscribed and circumscribed circles of a triangle.**
- **Construct a tangent line from a point outside a given circle to the circle.**

## WHAT I NEED TO KNOW:

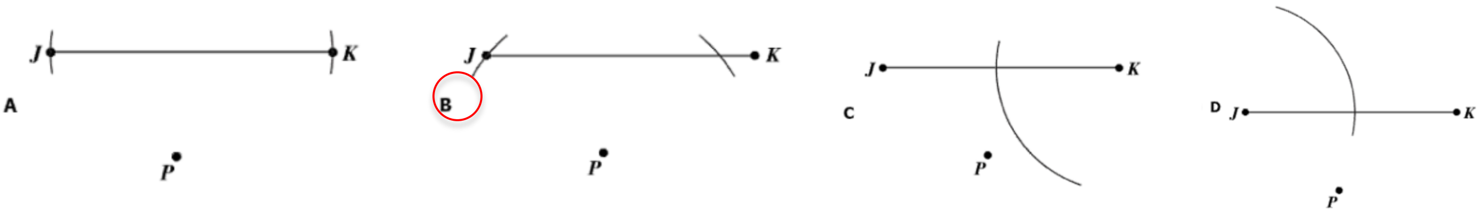
### WHAT EACH CONSTRUCTION LOOKS LIKE, THE STEPS, AND THE JUSTIFICATIONS

| CONSTRUCTION | STEPS | JUSTIFICATION |
|---|---|---|
|  | If a reference line does not already exist, draw a reference line with your straightedge upon which you will make your construction. Place a starting point on the reference line. Place the point of the compass on point *A*. Stretch the compass so that the pencil is exactly on *B*.<br>Without changing the span of the compass, place the compass point on the starting point on the reference line and swing the pencil so that it crosses the reference line. Label your copy. Your copy and (line segment) $\overline{AB}$ are congruent. | The two line segments are the same length, therefore they are congruent. |
|  | To draw an angle congruent to ∠A, begin by drawing a ray. Place the compass on point A and draw an arc across both sides of the angle. Without changing the compass radius, place the compass on endpoint of ray and draw a long arc crossing the ray. Set the compass so that its radius is BC. Place the compass on point where the arc crosses the ray and draw an arc intersecting the one drawn in the previous step. Use the straightedge to draw the second ray. | When this construction is finished, draw a line segment connecting where the arcs cross the sides of the angles. You now have two triangles that have 3 sets of congruent (equal) sides. SSS is sufficient to prove triangles congruent. Since the triangles are congruent, any leftover corresponding parts are also congruent - thus, the angle on the reference line and $\angle BAC$ are congruent. |
|  | Let point A be the vertex of the angle. Place the compass on point A and draw an arc across both sides of the angle. Place the compass on left intersection point and draw an arc across the interior of the angle. Without changing the radius of the compass, place it on right intersection point and draw an arc intersecting the one drawn in the previous step. Using the straightedge, draw ray bisecting ∠BAC. | To understand the explanation, some additional labeling will be needed. Label the point where the arc crosses side $\overline{AB}$ as *D*. Label the point where the arc crosses side $\overline{AC}$ as *E*. And label the intersection of the two small arcs in the interior as *F*. Draw segments $\overline{DF}$and $\overline{EF}$. By the construction, *AD* = *AE* (radii of same circle) and DF = EF (arcs of equal length). Of course *AF* = *AF*. All of these sets of equal length segments are also congruent. We have congruent triangles by SSS. Since the triangles are congruent, any of their leftover corresponding parts are congruent which makes $\angle BAF$equal (or congruent) to $\angle CAF$. |
|  | Begin with line segment AB. Place the compass at point A. Adjust the compass radius so that it is more than 1⁄2 the length of AB. Draw two arcs above and below the segment. Without changing the compass radius, place the compass on point B. Draw two arcs intersecting the previously drawn arcs. Using the straightedge, draw a line. Label the intersection point C. Point C is the midpoint of line segment AB, and the new line is perpendicular to line segment AB. | To understand the explanation you will need to label the point of intersection of the arcs above segment $\overline{AB}$ as *D* and below segment $\overline{AB}$ as *E*. Draw segments $\overline{AD}$, $\overline{AE}$, $\overline{BD}$ and $\overline{BE}$. All four of these segments are of the same length since they are radii of two congruent circles. More specifically, *DA* = *DB* and *EA* = *EB*. Now, remember a locus theorem: The locus of points equidistant from two points, is the perpendicular bisector of the line segment determined by the two points. Hence, $\overline{DE}$ is the **perpendicular** bisector of $\overline{AB}$. |
|  | Begin with point P, not on the line. Place the compass on point P. Using an arbitrary radius, draw arcs intersecting the line at two points. Place the compass at left intersection point. Adjust the compass radius so that it is more than 1⁄2 the length of the line. Draw an arc as shown here. Without changing the compass radius, place the compass on right intersection point. Draw an arc intersecting the previously drawn arc. Use the straightedge to draw line from P perpendicular to given line. | To understand the explanation, some additional labeling will be needed. Label the point where the arc crosses the line as points *C* and *D*. Label the intersection of the new arcs on the opposite side as point *E*. Draw segments $\overline{PC}$, $\overline{PD}$, $\overline{CE}$, and $\overline{DE}$. By the construction, *PC* = *PD* and *EC* = *ED*. Now, remember a locus theorem: The locus of points equidistant from two points (*C* and *D*),is the perpendicular bisector of the line segment determined by the two points. Hence, $\overline{PE}$ is the perpendicular bisector of $\overline{CD}$. |
|  | Begin with a line, containing point P. Place the compass on point P. Using an arbitrary radius, draw arcs intersecting the line at two points. Place the compass at left intersection point. Adjust the compass radius so that it is more than 1⁄2 the length of the line. Draw an arc. Without changing the compass radius, place the compass on the right intersection point. Draw an arc intersecting the previously drawn arc. Use the straightedge to draw the line perpendicular to point P. | Remember the construction for bisect an angle? In this construction, you have bisected the straight angle *P*. Since a straight angle contains 180 degrees, you have just created two angles of 90 degrees each. Since two right angles have been formed, a perpendicular exists. |
|  | With your straightedge, draw a transversal through point *P*. This is simply a straight line which runs through *P* and intersects the given line. Using your knowledge of the construction COPY AN ANGLE, construct a copy of the angle formed by the transversal and the given line such that the copy is located UP at point *P*. The vertex of your copied angle will be point P. When the copy of the angle is complete, you will have two parallel lines. This new line is parallel to the given line. | Since we used the construction to copy an angle, we now have two angles of equal measure in our diagram. In relation to parallel lines, these two equal angles are positioned in such a manner that they are called corresponding angles. A theorem relating to parallel lines tells us that if two lines are cut by a transversal and the corresponding angles are congruent (equal), then the lines are parallel. |

| CONSTRUCTION | STEPS | JUSTIFICATION |
|---|---|---|
|  | | |
|  | | |
|  | | |
|  | | |
|  | | |
|  | | |
|  | | |

# G.4 PROBLEMS:

**Which construction represents a correct first step in constructing a line segment perpendicular to $\overline{JK}$ through point $P$ ?**

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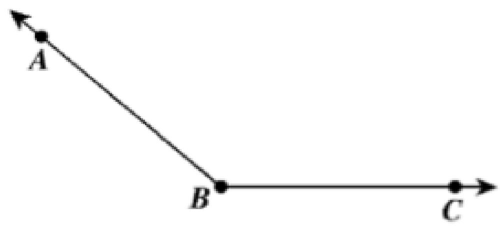
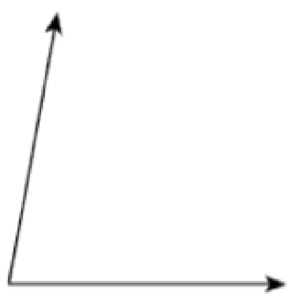
**Ben plans to bisect $\angle ABC$ to create the congruent angles $ABD$ and $CBD$.**

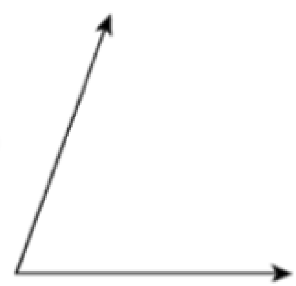



**Which angle is congruent to $\angle ABD$ and $\angle CBD$ ?**

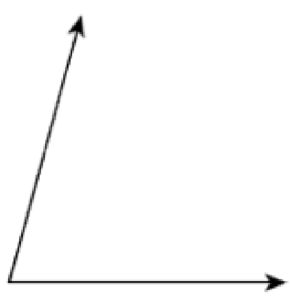
○ **A**



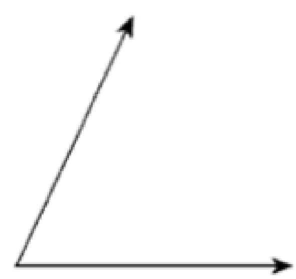
○ **C**



○ **B**



○ **D**



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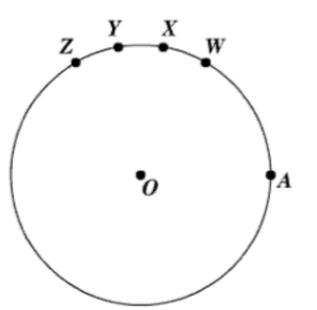
**Point $A$ represents a vertex of an equilateral triangle inscribed in circle $O$.**




**A** Point $W$

**B** Point $X$

**C** Point $Y$

**D** Point $Z$

**Which other point is also a vertex of this equilateral triangle?**

## SOL G.5

The student, given information concerning the lengths of sides and/or measures of angles in triangles, will

a) order the sides by length, given the angle measures;
b) order the angles by degree measure, given the side lengths;
c) determine whether a triangle exists; and
d) determine the range in which the length of the third side must lie.
These concepts will be considered in the context of real-world situations.

- Order the sides of a triangle by their lengths when given the measures of the angles.
- Order the angles of a triangle by their measures when given the lengths of the sides.
- Given the lengths of three segments, determine whether a triangle could be formed.
- Given the lengths of two sides of a triangle, determine the range in which the length of the third side must lie.
- Solve real-world problems given information about the lengths of sides and/or measures of angles in triangles.

## WHAT I NEED TO KNOW:

### HOW TO ORDER THE SIDES OF A TRIANGLE BY THEIR LENGTHS GIVEN THE MEASURES OF THE ANGLES

**In a triangle, the longest side is across from the largest angle.**

### HOW TO ORDER THE ANGLES OF A TRIANGLE BY THEIR MEASURES WHEN GIVEN THE LENGTHS OF THE SIDES

**In a triangle, the largest angle is across from the longest side.**

### HOW TO DETERMINE IF 3 GIVEN LENGTHS FORM A TRIANGLE

**The sum of the lengths of any two sides of a triangle must be greater than the third side.**
**Add the 2 smallest sides and see if the sum is greater than the greatest side.**

### HOW TO DETERMINE THE RANGE OF THE THIRD SIDE OF THE TRIANGLE GIVEN 2 SIDE LENGTHS

**Add the 2 sides to find the large number in the range. Subtract the 2 sides to find the small number in the range. Write the range as follows:**

**Small number < x < Large number**

**x represents possible side lengths of 3rd side of triangle**

## G.5 PROBLEMS:

**Given: Triangle $ABC$ with $AB = 42$ and $BC = 20$**

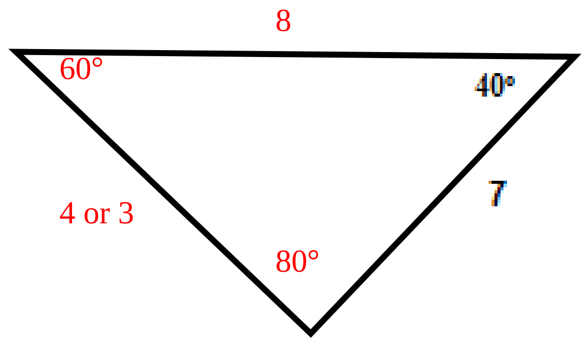
**Which of the following are possible lengths for $AC$?**

**12 20 22 32 42 50 62 70**

---

**Given the following diagram of a triangle, write in the angle measures and side lengths from the given box that would make the triangle possible. (Figure not drawn to scale.)**

| | | | |
|---|---|---|---|
| $100^o$ | $80^o$ | $60^o$ | 4 |
| 1 | 8 | $40^o$ | 3 |

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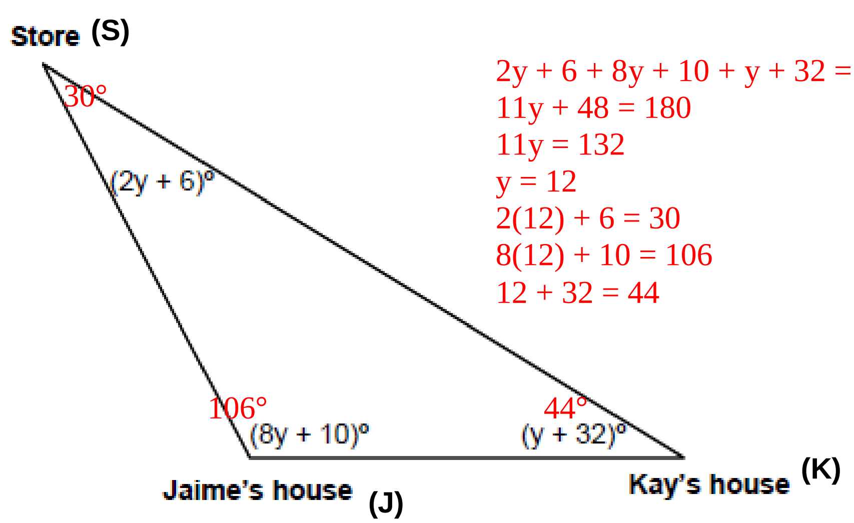
**Two sides of a triangle measure 9 inches and 13 inches. Write the numbers in the boxes that would correctly represent the range of the third side of the triangle.**

$4 < x < 22$

| | | | |
|---|---|---|---|
| 9 | 13 | -4 | 11 |
| | 21 | 5 | |

---

**The diagram is a map showing Jaime's house, Kay's house and the grocery store. Write the segments that represent the distances from each place in order from least to greatest.**




$\overline{JK}, \overline{SJ}, \overline{SK}$

$2y + 6 + 8y + 10 + y + 32 = 180$

$11y + 48 = 180$

$11y = 132$

$y = 12$

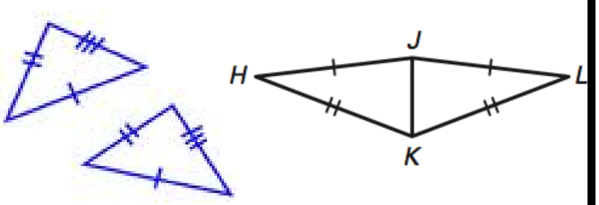
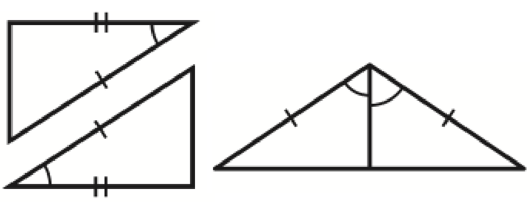
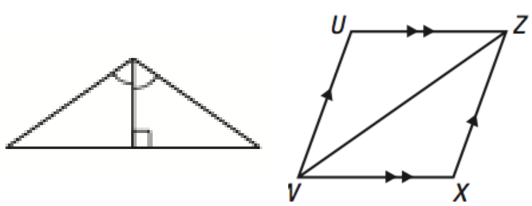
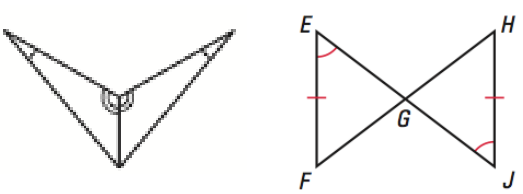
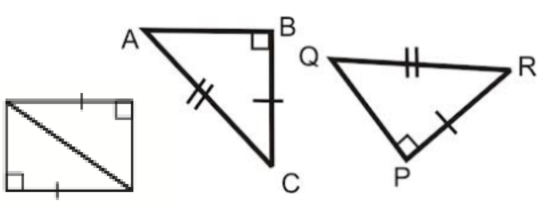
$2(12) + 6 = 30$

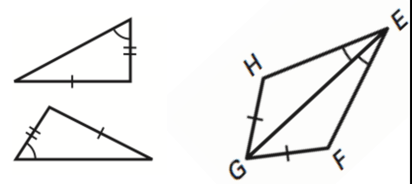
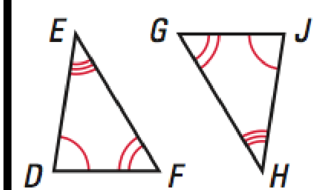
$8(12) + 10 = 106$

$12 + 32 = 44$

| SOL G.6 |
|---|
| The student, given information in the form of a figure or statement, will prove two triangles are congruent, using algebraic and coordinate methods as well as deductive proofs. |
| • Use definitions, postulates, and theorems to prove triangles congruent.<br>• Use coordinate methods, such as the distance formula and the slope formula, to prove two triangles are congruent.<br>• Use algebraic methods to prove two triangles are congruent. |

## WHAT I NEED TO KNOW:

| 5 METHODS OF PROVING TRIANGLES ARE CONGRUENT | |
|---|---|
|  | SSS |
|  | SAS |
|  | ASA |
|  | AAS |
|  | HL |

| 2 METHODS THAT DO NOT PROVE TRIANGLES ARE CONGRUENT | |
|---|---|
|  | ASS,<br>SSA |
|  | AAA |

| PROPERTIES THAT HELP PROVE TRIANGLES ARE CONGRUENT |
|---|
| Reflexive Property, Midpoint of a Segment, Symmetric Property, Transitive Property, Alternate Interior Angles, Corresponding Angles, Base Angles of Isosceles Triangle, Segment Bisector, Angle Bisector, Substitution Property |

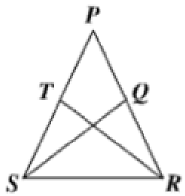
| FORMULA USED WHEN PROVING TRIANGLES ARE CONGRUENT USING COORDINATE GEOMETRY |
|---|
| Distance formula |

| WHAT DOES CPCTC MEAN? | HOW TO USE AND WRITE A ≅ STATEMENT |
|---|---|
| Corresponding Parts of Congruent Triangles are Congruent | $\Delta ABC \cong \Delta DEF$ means $\angle A \cong \angle D, \angle B \cong \angle E, \angle C \cong \angle F, \overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF},$ and $\overline{AC} \cong \overline{DF}$. |

## G.6 PROBLEMS:

**Select the reasons for the last three statements of this proof.**

Given: $\angle QSR \cong \angle TRS$; $\overline{PR} \cong \overline{PS}$



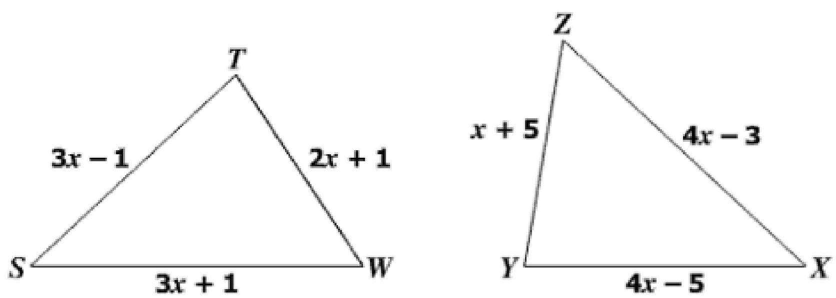

Prove: $\triangle QSR \cong \triangle TRS$

| Statements | Reasons |
|---|---|
| 1. $\overline{PR} \cong \overline{PS}$<br>$\angle QSR \cong \angle TRS$ | 1. Given |
| 2. $\angle TSR \cong \angle QRS$ | 2. Base angles of an isosceles triangle are congruent |
| 3. $\overline{SR} \cong \overline{RS}$ | 3. Reflexive Property |
| 4. $\triangle QSR \cong \triangle TRS$ | 4. Angle-Side-Angle (ASA) Postulate |

**Options**

- Base angles of an isosceles triangle are congruent
- Corresponding parts of congruent triangles are congruent
- Reflexive property
- Angle-Side-Angle (ASA) Postulate
- Side-Angle-Side (SAS) Postulate

**What value of $x$ makes $\triangle STW \cong \triangle XYZ$?**



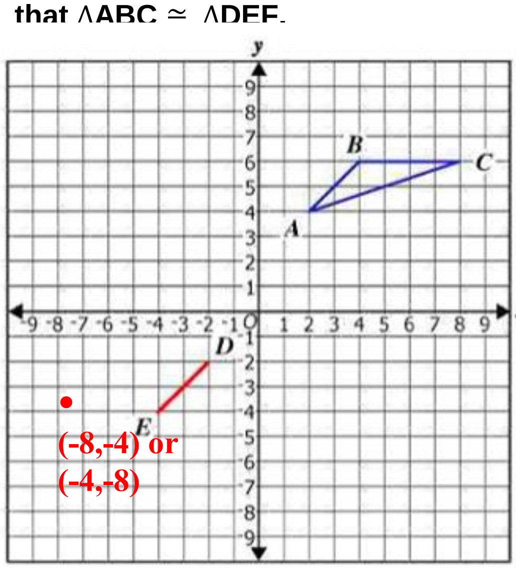

**A** 2

**B** 3

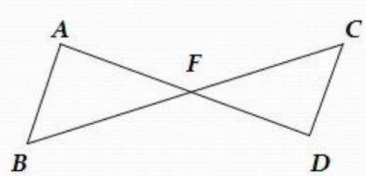
**C** 4

**D** 6

**The vertices of $\triangle ABC$ and the endpoints of $\overline{DE}$ have integral coordinates. Plot point F with integral coordinates so that $\triangle ABC \cong \triangle DEF$.**




Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AF} \cong \overline{FD}$



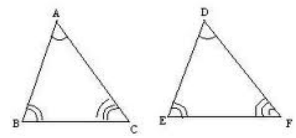
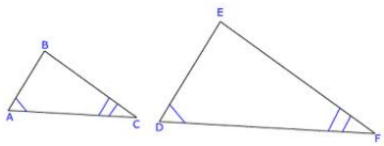
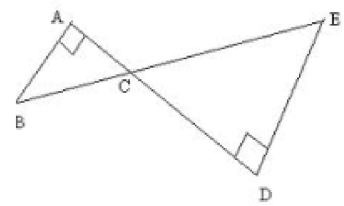
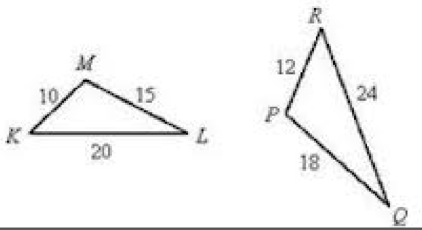
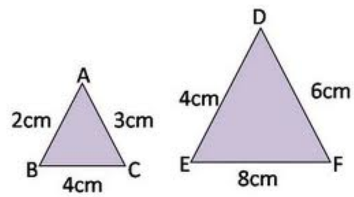
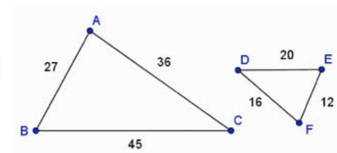
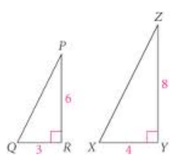
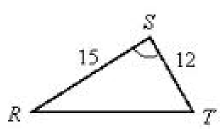
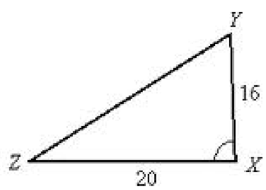
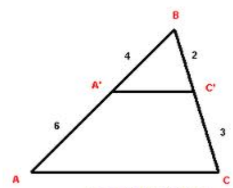

Prove: $\triangle$ ABF $\cong$ $\triangle$ DCF

| Statements | Reasons |
|---|---|
| 1. $\overline{AB} \parallel \overline{CD}$, $\overline{AF} \cong \overline{FD}$ | 1. Given |
| 2. $\angle BAF \cong \angle CDF$ | 2. If parallel lines are cut by a transversal, then pairs of alternate interior angles are congruent. |
| 3. $\angle AFB \cong \angle DFC$ | 3. Vertical angles are $\cong$. |
| 4. $\triangle ABF \cong \Delta$DCF | 4. Angle-Side-Angle (ASA) Postulate |

(other solutions are possible)

| SOL G.7 |
|---|
| The student, given information in the form of a figure or statement, will prove two triangles are similar, using algebraic and coordinate methods as well as deductive proofs. |
| • Use definitions, postulates, and theorems to prove triangles similar.<br>• Use algebraic methods to prove that triangles are similar.<br>• Use coordinate methods, such as the distance formula, to prove two triangles are similar. |

## WHAT I NEED TO KNOW:

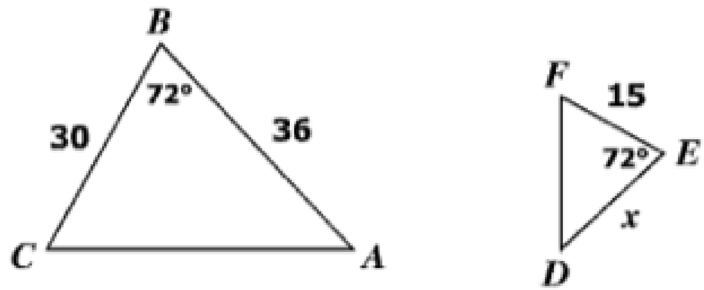
| 3 METHODS OF PROVING TRIANGLES ARE SIMILAR | |
|---|---|
|    | Angle-Angle<br>AA |
|    | Side-Side-Side<br>SSS |
|     | Side-Angle-Side<br>SAS |

| CORRESPONDING SIDES ARE PROPORTIONAL AND CORRESPONDING ANGLES ARE CONGRUENT |
|---|
| WHAT DOES THIS MEAN? |
| Corresponding sides are proportional means that if you find the scale factor of the corresponding longer sides, the scale factor of the corresponding smaller sides, and the scale factor of the corresponding medium sides, they should all be equivalent. Corresponding angles are congruent means that the angles in the same position in both triangles should have the same measure. |

| HOW TO SET UP A PROPORTION WHEN SOLVING FOR A MISSING SIDE | WHAT IS A SIMILARITY RATIO (SCALE FACTOR) AND HOW TO USE IT |
|---|---|
| $\frac{\text{Small side of } \triangle_1}{\text{Small side of } \triangle_2} = \frac{\text{Big side of } \triangle_1}{\text{Big side of } \triangle_2}$ | Scale factor = ratio of corresponding sides<br>$\frac{\text{Length of side of } \triangle_1}{\text{Corresponding length of side of } \triangle_2}$ |

## G.7 PROBLEMS:

**For what value of $x$ is $\triangle ABC \sim \triangle DEF$ ?**




A 18

B 21

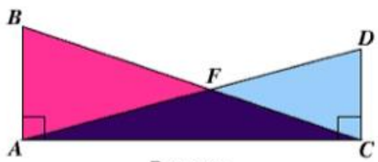
C 25

D 72

---

**Complete the proof.**

Given: $\overline{BA} \perp \overline{AC}$
$\overline{DC} \perp \overline{AC}$
Prove: $\triangle BFA \sim \triangle CFD$



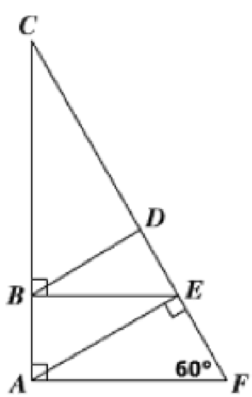

| Statements | Reasons |
|---|---|
| 1. Given: $\overline{BA} \perp \overline{AC}$, $\overline{DC} \perp \overline{AC}$ | 1. Given |
| 2. $\overline{BA} \parallel \overline{DC}$ | 2. If two lines are perpendicular to a third line, then the two lines are parallel. |
| 3. $\angle CDA \cong \angle BAD$; $\angle CBA \cong \angle BCD$ | 3. If two parallel lines are cut by a transversal, alternate interior angles are congruent. |
| 4. $\triangle BFA \sim \triangle CFD$ | 4. Angle-Angle (AA) Postulate |

- $\angle DFC \cong \angle BFA$; $\angle DAB \cong \angle BCD$
- $\angle CBA \cong \angle ADC$; $\angle BAD \cong \angle DCB$
- $\angle CDA \cong \angle BAD$; $\angle CBA \cong \angle BCD$
- Angle-Angle (AA) Postulate
- Side-Angle-Side (SAS) Postulate

---

**Given: $\triangle ACF$ is subdivided into smaller triangles**
**$\overline{AC} \perp \overline{AF}$ and $\overline{AC} \perp \overline{BE}$ and $\overline{AE} \perp \overline{CF}$**
**Point $B$ lies on $\overline{AC}$ and points $D$ and $E$ lie on $\overline{CF}$**

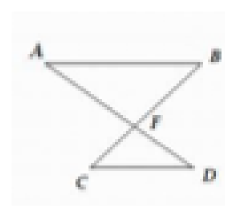
**Based on the given information, identify two triangles that may NOT be similar.**




$\triangle ACF$ | $\triangle BCE$ | $\triangle BEA$ | $\triangle DBE$ | $\triangle EAF$

Given: $\overline{AB} \parallel \overline{CD}$

Prove: $\Delta ABF \sim \Delta DCF$




| Statements | Reasons |
|---|---|
| 1. $\overline{AB} \parallel \overline{CD}$ | 1. Given |
| 2. $\angle BAF \cong \angle CDF$ | 2. Alternate interior angles are congruent. |
| 3. $\angle ABF \cong \angle DCF$ | 3. Vertical angles are congruent. |
| 4. . $\Delta ABF \cong \Delta DCF$ | 4. Angle-Angle (AA) Postulate |

# SOL G.8

The student will solve real-world problems involving right triangles by using the Pythagorean Theorem and its converse, properties of special right triangles, and right triangle trigonometry.

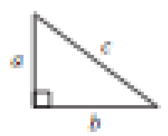
- Determine whether a triangle formed with three given lengths is a right triangle.
- Solve for missing lengths in geometric figures, using properties of 45°-45°-90° triangles.
- Solve for missing lengths in geometric figures, using properties of 30°-60°-90° triangles.
- Solve problems involving right triangles, using sine, cosine, and tangent ratios.
- Solve real-world problems, using right triangle trigonometry and properties of right triangles.
- Explain and use the relationship between the sine and cosine of complementary angles.

**CALCULATOR MODE**

degree

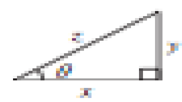
## WHAT I NEED TO KNOW:

## WHAT FORMULAS TO USE FROM THE FORMULA SHEET AND WHEN TO USE THEM




$$a^2 + b^2 = c^2$$

| WHEN TO: ADD | WHEN TO: SUBTRACT |
|---|---|
| Know legs, trying to find hypotenuse | Know hypotenuse and one leg, trying to find other leg |




$\text{Sin } \theta = \frac{y}{z}$

$\text{Cos } \theta = \frac{x}{z}$

$\text{Tan } \theta = \frac{y}{x}$

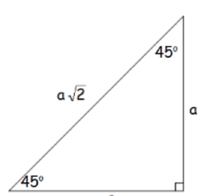
| WHEN TO: MULTIPLY | WHEN TO: DIVIDE | WHEN TO: USE INVERSE |
|---|---|---|
| Variable on top | Variable on the bottom | Trying to find the angle, $\vartheta$ |

## HOW TO DETERMINE WHETHER A TRIANGLE FORMED WITH THREE GIVEN LENGTHS IS A RIGHT TRIANGLE

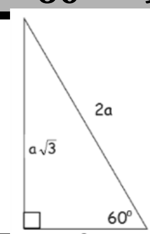
Use the converse of the Pythagorean theorem. Is $c^2$ equal to $a^2 + b^2$? C is the hypotenuse. A and B are the legs. Put $c^2$ in your calculator. Put $a^2 + b^2$ in your calculator. Check to see if they are equal.

## SPECIAL RIGHT TRIANGLES

$45° - 45° - 90°$




$30° - 60° - 90°$




## STEPS TO SOLVING FOR MISSING SIDE OR ANGLE USING TRIG. RATIOS

| | |
|---|---|
| STEP 1: | Circle the acute reference angle. ($\vartheta$) |
| STEP 2: | Label the 3 sides. H-hypotenuse, O-opposite side of $\vartheta$. A-adjacent side of $\vartheta$. Cross out which side does not have a partner. |
| STEP 3: | Choose trigonometric ratio based on given information and copy equation. |
| STEP 4: | Solve equation |

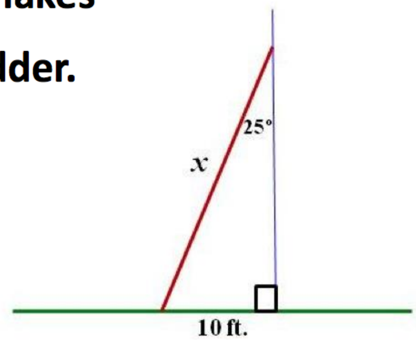
## HOW TO USE AND EXPLAIN THE RELATIONSHIP BETWEEN THE SINE AND COSINE OF COMPLEMENTARY ANGLES

The cosine of an acute angle of a right triangle is congruent to the sine of the complementary angle of that same triangle.

## G.8 PROBLEMS:

**A ladder leans against a wall. The bottom of the ladder is 10 feet from the base of the wall, and the top of the ladder makes an angle of 25° with the wall. Find the length, $x$, of the ladder.**

23.7 ft




---

**The figure represents the side view of a rectangular frame for metal shelves. Two diagonal braces support the frame.**

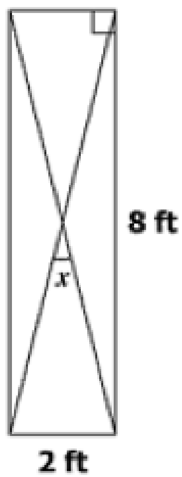
**Which is closest to the measure of $x$ ?**

**A** 7°

**B** 14°

**C** 28°

**D** 76°

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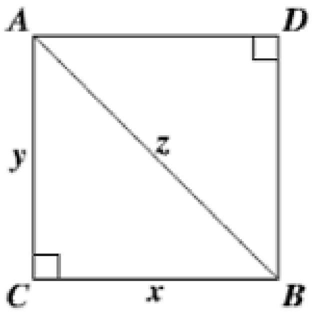
**This figure models a gate that has been constructed using two parallel vertical boards with a diagonal board connecting them. Identify all of the statements that must be true.**




$\sin\angle CAB + \cos\angle CAB = 180°$

$\sin\angle CAB = \cos\angle CBA$

$\angle CAB \cong \angle DAB$

$x^2 + y^2 = z^2$

$\overline{AD} \parallel \overline{CB}$

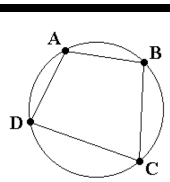
| SOL G.9 |
|---|
| The student will verify characteristics of quadrilaterals and use properties of quadrilaterals to solve real-world problems. |
| • Solve problems, including real-world problems, using the properties specific to parallelograms, rectangles, rhombi, squares, isosceles trapezoids, and trapezoids.<br>• Prove that quadrilaterals have specific properties, using coordinate and algebraic methods, such as the distance formula, slope, and midpoint formula.<br>• Prove the characteristics of quadrilaterals, using deductive reasoning, algebraic, and coordinate methods.<br>• Prove properties of angles for a quadrilateral inscribed in a circle. |

## WHAT I NEED TO KNOW:

| CHARACTERISTICS | | WHICH QUADRILATERALS HAVE THIS CHARACTERISTIC | WHAT ARE THE COORDINATE PROOFS FOR THIS CHARACTERISTIC |
|---|---|---|---|
| **SIDES** | | | |
| 1. | Opp. sides ∥ | Parallelogram, rectangle, square, rhombus | Slope formula |
| 2. | Opp. sides ≅ | Parallelogram, rectangle, square, rhombus | Distance formula |
| 3. | Exactly one pair of opp. sides ∥ | trapezoid | Slope formula |
| **ANGLES** | | | |
| 1. | Opp. angles ≅ | Parallelogram, rectangle, square, rhombus | |
| 2. | Consecutive angles supplementary | Parallelogram, rectangle, square, rhombus | |
| **DIAGONALS** | | | |
| 1. | Diagonals bisect each other | Parallelogram, rectangle, square, rhombus | Midpoint formula |
| 2. | Diagonals are congruent | Rectangle, Square, Isosceles Trapezoid | Distance formula |
| 3. | Diagonals are perpendicular | Rhombus, Square, Kite | Slope formula |
| 4. | Diagonals bisect the angles | Rhombus, Square | |

## HOW TO PROVE A QUADRILATERAL IS A SPECIFIC FIGURE

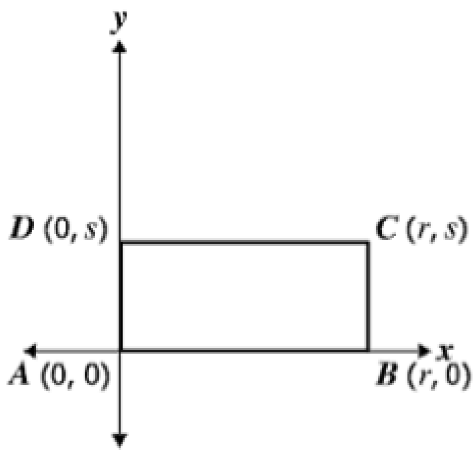
| PARALLELOGRAM | RECTANGLE | RHOMBUS | SQUARE | ISOSCELES TRAPEZOID | TRAPEZOID |
|---|---|---|---|---|---|
| Prove opposite sides are ≅ | Prove opp. sides are ≅ and diagonals ≅ | Prove all sides are ≅ | Prove all sides ≅ and four right ∠'s | Prove one pair of sides is ∥ and other is not and prove non ∥ sides are ≅ | Prove one pair of sides is ∥ |




| PROPERTIES OF ANGLES FOR A QUAD INSCRIBED IN A CIRCLE | Opposite angles are supplementary |
|---|---|

G.9 PROBLEMS:

**Given: Quadrilateral $ABCD$**




**Which expression proves that $ABCD$ is a rectangle?**

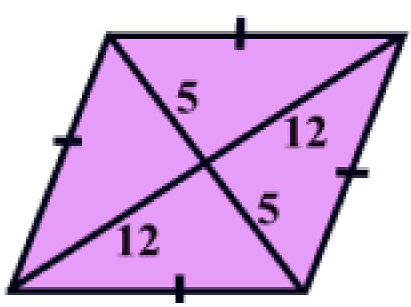
A The length of each diagonal is $\sqrt{r^2 + s^2}$.

B The common midpoint of the diagonals is $\left(\frac{r}{2}, \frac{s}{2}\right)$.

C The slope of $\overline{AC}$ is $\frac{s}{r}$ and the slope of $\overline{BD}$ is $^{-}\frac{s}{r}$.

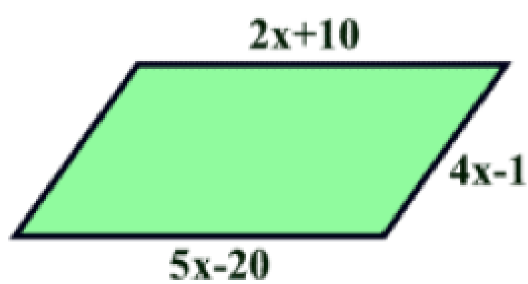
D The length of both $\overline{AB}$ and $\overline{CD}$ is $r$ and the length of both $\overline{AD}$ and $\overline{BC}$ is $s$.

The diagonals of a rhombus are 10 and 24. Find the perimeter of the rhombus.




13(4) = 52

**Find the length of the side of the parallelogram represented by 4x -1.**




2x + 10 = 5x – 20
x = 10
4x – 1 = 39

## SOL G.10

**The student will solve real-world problems involving angles of polygons.**

- Solve real-world problems involving the measures of interior and exterior angles of polygons.
- Identify tessellations in art, construction, and nature.
- Find the sum of the measures of the interior and exterior angles of convex polygon.
- Find the measure of each interior and exterior angle of a regular polygon.
- Find the number of sides of a regular polygon, given the measures of interior or exterior angles of the polygon.

## WHAT I NEED TO KNOW:

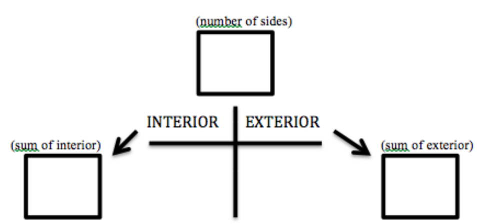
### POLYGONS

#### DEFINITION OF REGULAR POLYGON

Polygon with all sides congruent and all angles congruent.

#### NUMBER OF SIDES GIVEN THE NAME

| TRIANGLE | QUADRILATERAL | PENTAGON | HEXAGON | HEPTAGON | OCTAGON | NONAGON | DECAGON | DODECAGON |
|---|---|---|---|---|---|---|---|---|
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 12 |

| HOW TO FIND THE SUM OF THE INTERIOR AND EXTERIOR OF CONVEX POLYGONS | HOW TO FIND THE MEASURE OF EACH INTERIOR AND EXTERIOR OF A REGULAR POLYGON | HOW TO FIND THE NUMBER OF SIDES OF A REGULAR POLYGON, GIVEN THE MEASURES OF INTERIOR OR EXTERIOR ANGLES OF THE POLYGON |
|---|---|---|
| Sum of interior = (n-2) · 180<br><br>Sum of exterior = $360^0$ | Exterior = 360/ # of sides<br>Interior = 180 - exterior<br><br> | Given interior: 180 – interior = exterior<br>360/exterior = # of sides<br>Given exterior: 360/exterior = # of sides |

### TESSELLATIONS

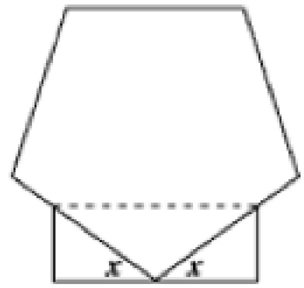
#### WHICH REGULAR POLYGONS TESSELLATE

| Equilateral Triangle | Square | Hexagon |
|---|---|---|

| DEFINITION OF A TESSELLATION | THE RULE ABOUT INTERIOR ANGLES |
|---|---|
| A repeated pattern with no gaps or overlaps that covers a plane | Only regular polygons that have an interior angle which is a factor of 360 will tessellate. |

## G.10 PROBLEMS:

This figure is composed of a regular pentagon and a rectangle.




What is the measure of each of the angles identified as $x$ ?

A 36°

B 54°

C 72°

D 108°

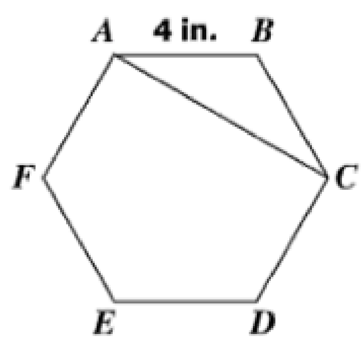
---

**Which of these regular polygons could tessellate a plane?**

**Square** **Pentagon** **Octagon** **Hexagon** **Decagon**

---

The figure shown is a regular hexagon.




What is the length of the diagonal $AC$ ?

A $4\sqrt{3}$ in.

B 8 in.

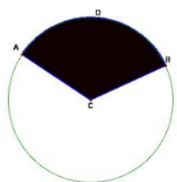
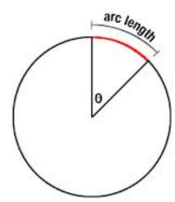
C 12 in.

D $8\sqrt{3}$ in.

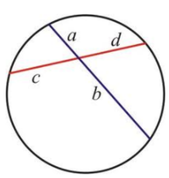
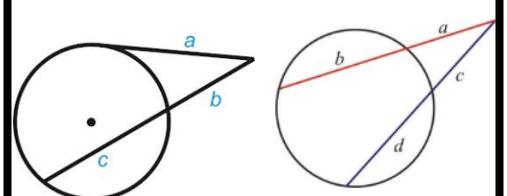
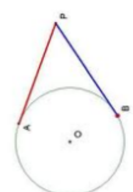
| SOL G.11 |
|---|
| The student will use angles, arcs, chords, tangents, and secants to<br>a) investigate, verify, and apply properties of circles;<br>b) solve real-world problems involving properties of circles; and<br>c) find arc lengths and areas of sectors in circles. |
| • Find lengths, angle measures, and arc measures associated with<br>– two intersecting chords;<br>– two intersecting secants;<br>– an intersecting secant and tangent;<br>– two intersecting tangents; and<br>– central and inscribed angles.<br>• Calculate the area of a sector and the length of an arc of a circle, using proportions.<br>• Solve real-world problems associated with circles, using properties of angles, lines, and arcs.<br>• Verify properties of circles, using deductive reasoning, algebraic, and coordinate methods. |

## WHAT I NEED TO KNOW:

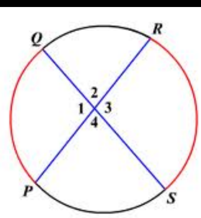
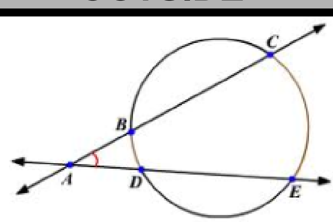
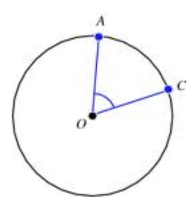
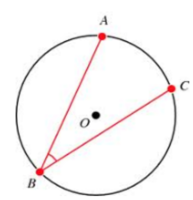
**FORMULAS** (AREA AND CIRCUMFERENCE ON FORMULA SHEET)

| AREA OF SECTOR | | LENGTH OF AN ARC | |
|---|---|---|---|
|  | Part of area<br>$\frac{m^o \text{ of arc}}{360^o} \cdot \pi r^2$ |   | Part of circumference<br>$\frac{m^o \text{ of arc}}{360^o} \cdot 2\pi r$ |

**LENGTHS**

| INTERSECTING INSIDE OF CIRCLE | | INTERSECTING OUTSIDE OF CIRCLE | | | |
|---|---|---|---|---|---|
|   | PP = PP<br>*ab = cd* |   | OW = OW<br>*aa = b(b+c)*<br>*a(a+b)=c(c+d)* |  | conehead<br>AC = AB |

**ANGLE MEASURES**

| INSIDE | OUTSIDE | CENTRAL | ON |
|---|---|---|---|
|   |   |   |   |
| Angle = $\frac{\text{arc + arc}}{2}$ | Angle = $\frac{\text{arc – arc}}{2}$ | Angle = arc | Angle = $\frac{\text{arc}}{2}$ |

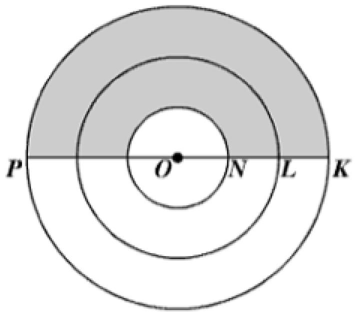
| CONGRUENT CHORDS | ANGLE BETWEEN TANGENT AND RADIUS |
|---|---|
| If chords are congruent, then the intercepted arcs are congruent. | The angle between the tangent and the radius is $90^o$ |

# G.11 PROBLEMS:

**Given: Three concentric circles with the center $O$**

$\overline{KL} \cong \overline{LN} \cong \overline{NO}$

$KP = 42$ **inches**




**Which is closest to the area of the shaded region?**

**A** 231 sq in.

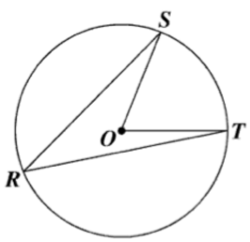
**B** 308 sq in.

**C** 539 sq in.

**D** 616 sq in.

---

**In circle $O$, $m\angle SOT = 68°$.**




**What is $m\angle SRT$ ?**

$m\angle SRT =$ 34 °

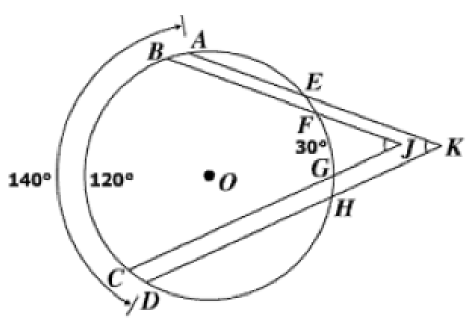
---

**Bob divides his circular garden into 10 congruent sectors to plant different types of flowers. The circumference of Bob's garden is 50.5 feet. What is the area of one sector of Bob's garden?**

20.3 sq. ft.

---

**In circle $O$, $m\widehat{FG} = 30°$, $m\widehat{BC} = 120°$, and $\angle J \cong \angle K$.**




**What is $m\widehat{EH}$ ?**

**A** 35°

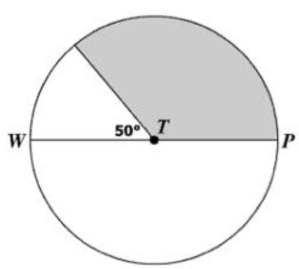
**B** 40°

**C** 45°

**D** 50°

---

**Given: Circle $T$ with $WP = 36$ centimeters**




**Which best represents the area of the shaded sector?**

**A** $117\pi$ cm$^2$

**B** $180\pi$ cm$^2$

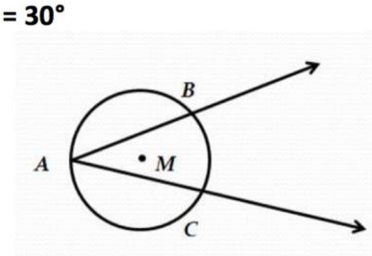
**C** $234\pi$ cm$^2$

**D** $468\pi$ cm$^2$

---

**Given: Circle $M$ with secants $\overrightarrow{AB}$ and $\overrightarrow{AC}$**

**$m\angle A = 30°$**




**If the length of arc $BC$ is 3 cm, what is the circumference of the circle?**

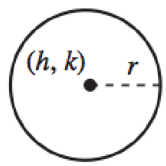
18 cm

| SOL G.12 |
| --- |
| The student, given the coordinates of the center of a circle and a point on the circle, will write the equation of the circle. |
| • Identify the center, radius, and diameter of a circle from a given standard equation.<br>• Use the distance formula to find the radius of a circle.<br>• Given the coordinates of the center and radius of the circle, identify a point on the circle.<br>• Given the equation of a circle in standard form, identify the coordinates of the center and find the radius of the circle.<br>• Given the coordinates of the endpoints of a diameter, find the equation of the circle.<br>• Given the coordinates of the center and a point on the circle, find the equation of the circle.<br>• Recognize that the equation of a circle of given center and radius is derived using the Pythagorean Theorem. |

## WHAT I NEED TO KNOW:

| HOW TO USE THE DISTANCE FORMULA TO FIND THE RADIUS OF THE CIRCLE |
| --- |
| Given the center and a point on the circle, the radius is the distance from the center to the point |

**ON FORMULA SHEET:**




$$(x-h)^2 + (y-k)^2 = r^2$$

**DERIVED FROM PYTHAGOREAN THEOREM!!!**

| HOW TO IDENTIFY THE FOLLOWING GIVEN EQUATION: | | |
| --- | --- | --- |
| **CENTER** | **RADIUS** | **DIAMETER** |
| (h, k) | r | Diameter = 2r |

| HOW TO FIND THE EQUATION OF THE CIRCLE GIVEN: | |
| --- | --- |
| **COORDINATES OF THE ENDPOINTS OF A DIAMETER** | **COORDINATES OF THE CENTER AND A POINT ON THE CIRCLE** |
| Use the distance formula to find the length of diameter.<br>Radius = diameter/2<br>Use midpoint formula to find the coordinate of the center (h,k).<br>Plug h, k, and r into the equation for equation of a circle on formula sheet. | Use distance formula to find length of radius.<br>Use center (h,k) and radius, r, and plug into formula of equation of a circle on formula sheet. |

| HOW TO IDENTIFY A POINT ON THE CIRCLE GIVEN THE CENTER AND RADIUS | |
| --- | --- |
| **WITHOUT CALCULATOR** | **WITH CALCULATOR APP** |
| On graph paper, plot the center of the circle.<br>From the center, count, either up, down, left or right the length of the radius. | Go to APPS<br>CONICS<br>ENT ENT<br>Type in h, k, and r<br>GRAPH<br>TRACE<br>Right arrow key will give different points |

## G.12 PROBLEMS:

Given: Circle $O$ with diameter $\overline{CD}$
$C(^-7, ^-4)$ and $D(1, 2)$

Create the equation of this circle.

The Equation of the Circle

$(x + 3)^2 + (y + 1)^2 = 25$

| | |
|---|---|
| $(x - 3)^2$ | $(x + 3)^2$ |
| $(y - 1)^2$ | $(y + 1)^2$ |
| 25 | 100 |

Given: Circle $W$
$W(^-4, 6)$
Radius = 10 units

Which point lies on circle $W$?

A (0, 4)

B (2, 10)

C (4, 0)

D (6, 16)

---

The coordinates of the center of a circle are (-2,6). This circle has a diameter of 10 units.

a) What is the equation of the circle? $(x + 2)^2 + (y - 6)^2 = 25$

b) Give the integral coordinates of two points that lie on the circle.

Possible points: (-2,1), (-2,11), (-7,6), (3,6)

---

The equation of a circle is $(x - 3)^2 + (y + 4)^2 = 16$.

a) What are the coordinates of the center of the circle? (3,-4)

b) What is the radius of the circle? 4

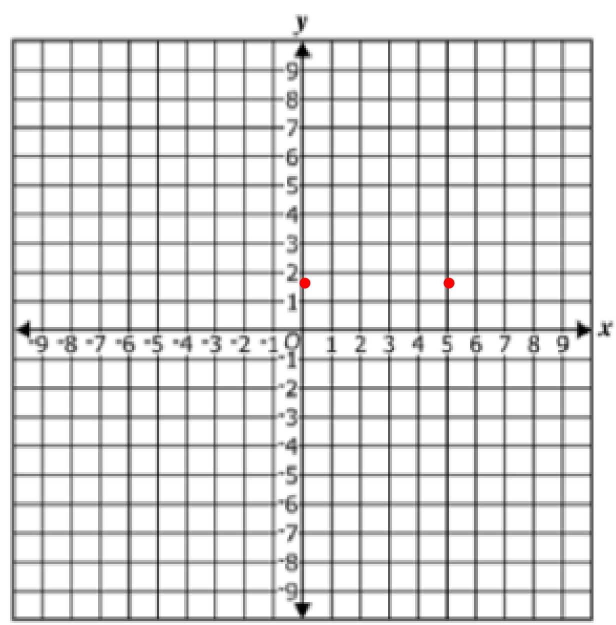
c) What is the diameter of the circle? 8

d) Give the integral coordinates of two points that lie on the circle.

Possible points: (-1,-4), (7,-4), (3,-8), (3,0)

---

Circle *O* is defined by the equation $x^2 + (y - 2)^2 = 25$. Plot the center of circle *O* and one point with integral coordinates that lies on circle *O*.



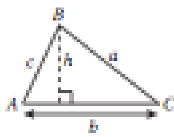
# SOL G.13

**The student will use formulas for surface area and volume of three-dimensional objects to solve real-world problems.**

- **Find the total surface area of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.**
- **Calculate the volume of cylinders, prisms, pyramids, cones, and spheres, using the appropriate formulas.**
- **Solve problems, including real-world problems, involving total surface area and volume of cylinders, prisms, pyramids, cones, and spheres as well as combinations of three-dimensional figures.**
- **Calculators may be used to find decimal approximations for results.**

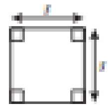
## WHAT I NEED TO KNOW:

## HOW TO USE THE FORMULAS ON THE FORMULA SHEET

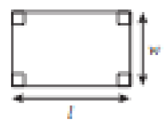


$A = \frac{1}{2}bh$

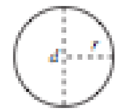
$A = \frac{1}{2}ab \sin C$



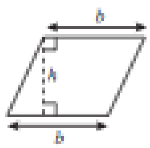
$p = 4s$
$A = s^2$



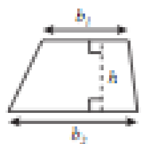
$p = 2l + 2w$
$A = lw$



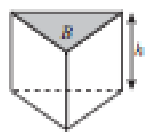
$C = 2\pi r$
$C = \pi d$
$A = \pi r^2$



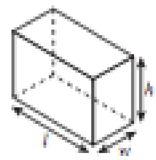
$A = bh$



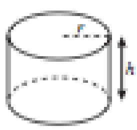
$A = \frac{1}{2}h(b_1 + b_2)$



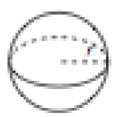
$V = Bh$
$L.A. = hp$
$S.A. = hp + 2B$



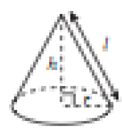
$V = lwh$
$S.A. = 2lw + 2lh + 2wh$



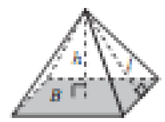
$V = \pi r^2 h$
$L.A. = 2\pi rh$
$S.A. = 2\pi r^2 + 2\pi rh$



$V = \frac{4}{3}\pi r^3$
$S.A. = 4\pi r^2$



$V = \frac{1}{3}\pi r^2 h$
$L.A. = \pi rl$
$S.A. = \pi r^2 + \pi rl$



$V = \frac{1}{3}Bh$

$L.A. = \frac{1}{2}lp$

$S.A. = \frac{1}{2}lp + B$

## KEY WORDS FOR VOLUME

Units cubed
Amount of
space occupied
by object

## THE DIFFERENCE BETWEEN B AND b

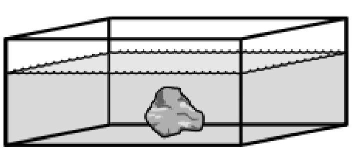
**B means area of the base.**
**b means length of base.**

**WATCH YOUR UNITS!!!**

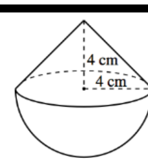
## KEY WORDS FOR SURFACE AREA

**Units squared**
**How much material needed to cover…**

## SOMETIMES YOU MAY NEED TO ADD VOLUMES AND SOMETIMES YOU MAY NEED TO SUBTRACT!



**Volume of water with rock minus volume of water = volume of rock**




**Volume of cone plus volume of half of sphere = total volume**

## G.13 PROBLEMS:

**A cylinder has a volume of $300\pi$ cubic centimeters and a base with a circumference of $10\pi$ centimeters. What is the height of the cylinder?**

**A** 30 cm

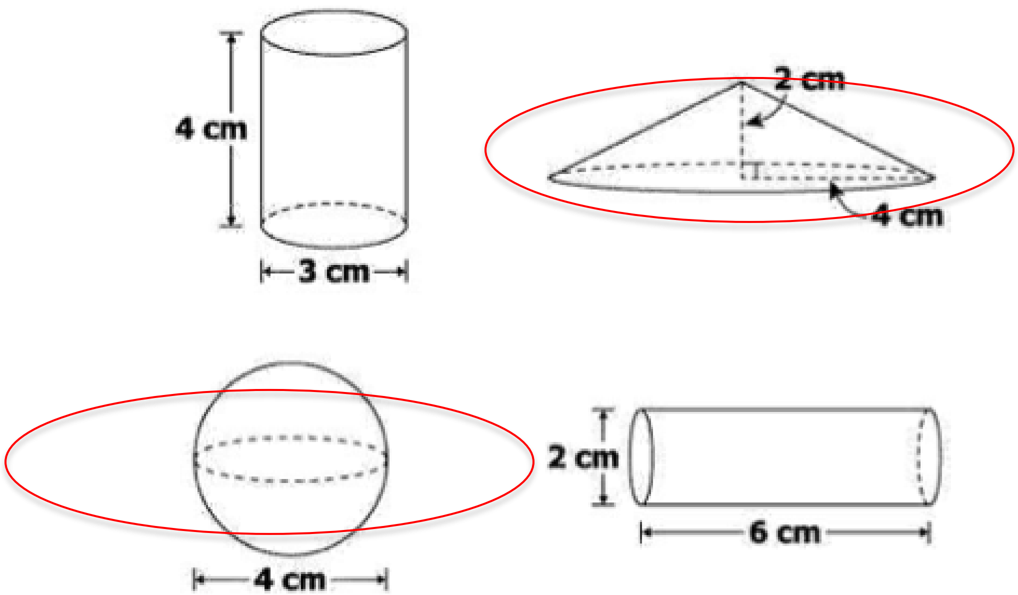
**B** 15 cm

**C** 12 cm

**D** 3 cm

---

**Two cylinders, a sphere, and a cone are shown. Select the two objects with the same volume.**

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**A cone has a slant height of 10 centimeters and a lateral area of $60\pi$ square centimeters. What is the volume of a sphere with a radius equal to that of the cone?** 904.32 $cm^3$

---

**A fish tank in the shape of a rectangular prism has these dimensions:**

- **length = 20 inches**
- **width = 10 inches**
- **height = 12 inches**

**What is the volume of water in the tank when it is $\frac{4}{5}$ full?**

1920 $in^3$

## SOL G.14

The student will use similar geometric objects in two- or three-dimensions to
a) compare ratios between side lengths, perimeters, areas, and volumes;
b) determine how changes in one or more dimensions of an object affect area and/or volume of the object;
c) determine how changes in area and/or volume of an object affect one or more dimensions of the object; and
d) solve real-world problems about similar geometric objects.

- **Compare ratios between side lengths, perimeters, areas, and volumes, given two similar figures.**
- **Describe how changes in one or more dimensions affect other derived measures (perimeter, area, total surface area, and volume) of an object.**
- **Describe how changes in one or more measures (perimeter, area, total surface area, and volume) affect other measures of an object.**
- **Solve real-world problems involving measured attributes of similar objects.**

## WHAT I NEED TO KNOW:

| RATIOS | | | |
|---|---|---|---|
| **LENGTHS** | **PERIMETERS** | **SURFACE AREA** | **VOLUME** |
| $\frac{a}{b}$ | $\frac{a}{b}$ | $\frac{a^2}{b^2}$ | $\frac{a^3}{b^3}$ |

## DETERMINE HOW CHANGES IN ONE OR MORE DIMENSIONS OF AN OBJECT AFFECT AREA/OR VOLUME OF THE OBJECT AND VICE-VERSA

**Use your own numbers to fit situation. Plug in numbers to first situation. Then plug in numbers to the change of situation. Observe the change.**

## G.14 PROBLEMS:

A company makes two similar cylindrical containers. The total surface area of the smaller container is 0.81 times that of the larger container. The height of the larger container is 60 centimeters. What is the height of the smaller container?

A 54 cm

B 48.6 cm

C 24.3 cm

D 21 cm

---

A rectangular prism has a volume of 36 $cm^3$.

a) If the height of the prism is tripled and the other dimensions do not change, what is the volume of the new rectangular prism? 108 $cm^3$

b) If all dimensions of the original rectangular prism are tripled, what is the volume of the new rectangular prism? 972 $cm^3$

---

The ratio of the volume of two spheres is 8:27. What is the ratio of the lengths of the radii of these two spheres?

2 : 3

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 6 | 8 | 9 | 13 | 19 | 27 |

---

A cylinder has a surface area of 96 square inches. If all dimensions of this cylinder are multiplied by $\frac{1}{2}$ to create a new cylinder, what will be the surface area of the new cylinder?

24 square inches

---

The heights of two similar triangles are in the ratio 2:5. If the area of the larger triangle is 400 square units, what is the area of the smaller triangle?

A 64 square units

B 160 square units

C 1,000 square units

D 2,500 square units

---

If the height of a rectangular prism is decreased by $\frac{1}{3}$, then which statement is true?

A The volume would decrease by $\frac{1}{3}$.

B The volume would decrease by $\frac{1}{6}$.

C The volume would decrease by $\frac{1}{9}$.

D The volume would decrease by $\frac{1}{27}$.